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Ing. Daniel Vozák

Summary of doctoral dissertation

ROBUST PREDICTIVE CONTROL OF LINEAR SYSTEMS

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Author: Ing. Daniel Vozák
Institute of Robotics and Cybernetics, Faculty of Electrical Engineering and Information Technology, Ilkovičova 3, 812 19 Bratislava

Supervisor: prof. Ing. Vojtech Veselý, DrSc.
Institute of Robotics and Cybernetics, Faculty of Electrical Engineering and Information Technology, Ilkovičova 3, 812 19 Bratislava

Opponents: prof. Ing. Boris Rohaľ-Ilkiv, CSc.
Institute of automation, measurement and applied informatics, Faculty of mechanical Engineering, Nám. slobody 17, 812 31 Bratislava 1

prof. Ing. Aleš Janota, PhD.
Department of Control and Information Systems, Faculty of Electrical Engineering, University of Žilina, Univerzitna 1, 010 26 Žilina

Thesis summary was submitted on ....

Date and location of PhD thesis defence: ....at Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Ilkovičova 3, 812 19 Bratislava.

prof. RNDr. Gabriel Juhás, PhD.
Dean of FEI STU

Abstract

Thesis Tittle: Robust predictive control of linear systems

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This thesis deals with the robust predictive controller design. Robust stability of the closed-loop is important in the real systems where the parameters are uncertain and the designer must be aware of the uncertainty in the controller design procedure. This is a well known problem in the standard predictive controller because it does not guarantee the closed-loop stability. This work focuses only on the polytopic uncertainty description. Two robust predictive controller algorithms are presented. First is based on the transfer function model of the system and is suitable for SISO system. The method uses off-line minimization of the sum of squares of errors for the controller parameters calculation and it adds a variable gain approach to achieve input constrains. Second algorithm uses state-space model and is based on the previous works. The design procedure is replaced with a less conservative parameter dependent Lyapunov function. The main contribution is in the practical implementation of the algorithm where we made several improvements. The controller is augmented with the set-point and the derivative part. Two realization schemes are created and a new feedback structure is proposed which allows to separate prediction horizons for controller realization and practical implementation. Moreover, this approach creates a new possibilities for constraints handling with several feedback gains. All results are experimentally proved on simulations and real systems.

Anotácia dizertačnej práce

Názov dizertačnej práce: Robustné prediktívne riadenie lineárnych systémov

Kľúčové slová: Prediktívne riadenie, Robustné riadenie, Polytopický model, Obmedzenie akčného zásahu

Práca sa zaoberá návrhom robustných prediktívnych regulátorov. Robustná stabilita uzavretého regulačného obvodu je dôležitou vlastnosťou pri reálnych systémoch a je potrebné ju zahrnúť do návrhu regulátora. Zároveň je to známy problém štandardných prediktívnych regulátorov, ktoré negarantujú stabilitu systému. Táto práca je zameraná na polytopický opis neurčitosti. Navrhnuté sú dva algoritmy robustných prediktívnych regulátorov. Prvý vychádza z modelu systému v tvare prenosovej funkcie a je vhodný pre SISO systémy. Metóda návrhu používa minimalizáciu sumy štvorcov odchýlok na výpočet parametrov regulátora a variabilné zosilnenie na zabezpečenie obmedzenia akčného zásahu. Druhá metóda používa stavový model systému a je založená na predchádzajúcich prácach. Výpočet regulátora je nahradený menej konzervatívnym s použitím parametricky závislej ljapunovovej funkcie. Hlavným prínosom je praktická realizácia algoritmu s viacerými zlepšeniami. Regulátor je doplnený žiadanou hodnotou a derivačnou zložkou. Vytvorené sú dve schémy na realizáciu a navrhnutá je nová štruktúra výstupnej spätnej väzby, ktorá umožňuje oddeliť horizont predikcie pre výpočet optimálnych parametrov a horizont pre realizáciu a obmedzenia. Navyše, tento prístup poskytuje nové možnosti pri obmedzení akčného zásahu s viacerými rôznymi zosilneniami spätnej väzby. Všetky výsledky sú experimentálne overené na simuláciach a aj reálnych systémoch.

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1 Introduction

The linear control theory became very well developed in the last century. Controller designers can use an immense number of methods to find the best controller parameters starting from simple experimental methods and loop shaping techniques or numerical methods to more advanced optimal control. When it comes to extend the standard linear control theory there are two main areas which became the most important.

First is the linear system with constraint. Every real system contains some constraints and it is necessary to take account of them in the controller design procedure or even in the controller algorithm. Thus, it is a natural way how to expand standard linear models. The basic solution of this problem was to include some anti-windup techniques. More systematic approach which allowed new possibilities is the model predictive control (MPC). That is why MPC attracted a lot of practitioners and became one of the most used advanced control techniques in the industrial applications. There exists several MPC formulations based on the state-space, transfer function or step/impulse models. All of them are non-linear controllers which makes the analysis of the closed-loop properties much harder. The underlying idea of MPC is to use the system model to predict the future system behaviour and then to find an optimal system input by minimization of a cost function.

Second area is the robust control theory which allows to design a controller with guaranteed stability and performance for the system model with defined uncertainty. This approach is necessary because it is impossible to obtain a perfect model and/or the system parameters can vary depending on some working aspects. The uncertainty can be defined in various forms like the interval model, the polytopic model, additive/multiplicative uncertainty... There exists a wide range of methods for the robust stability analysis for both SISO and MIMO systems. They differs not only in the type of uncertainty but also in conservativeness. Nowadays, we have robust controller design procedures which can reliably guarantee the robust closed-loop stability and performance.

The predictive control and robust control are two different approaches that augment linear control theory with new possibilities and advantages but still hold some parts of the well developed linear system theory. Therefore, we do not need to use non-linear systems for the closed-loop system analysis or the controller design. A lot of research was done to connect the guaranteed stability and robust stability with optimality and constraints handling of predictive control. Although, it yielded plenty of scientific publications there are still open problem which needs to be solved.

2 Problems in the MPC algorithms

The MPC controllers are interesting for industrial applications because they bring some additional advantages over classical PID control which allows to have better performance or/and more economically effective production. It is mainly the ability of MPC to

- handle system constraint and anti-windup problems,
- control large MIMO systems,
- and optimality of the controller with simple tuning by weighting matrices in the cost function.

Although, the MPC was successfully applied to a wide range of industrial processes it contains some limitations which are caused by the drawbacks in the MPC formulation. In the standard MPC without modifications it is

- the closed-loop stability is not guaranteed,
- MPC does not include any robust stability and performance,
- computational complexity of QP solver in each sample time
- and the feasibility of the cost function with constraints.

The problem of stability is tightly connected with the robust stability problem. It is well known that all system models are imperfect and contains some uncertainty. So, the design of a controller with guaranteed stability in the whole uncertainty is fundamental for a successful application on a real plant. Thus, the lack of stability and robust stability is considered to be the most severe drawback of MPC. In the literature we can find two basic principles of the most used modifications how to ensure stability in MPC

- Terminal constraint in the cost function
- Infinite time horizon

Both principles can guarantee the closed-loop stability but the the on-line computational complexity increases and the optimization becomes more often infeasible. The on-line computational complexity was highly reduced in the explicit MPC. It does not change the cost function but only modifies the practical implementation of the predictive control algorithm. It shows the control action depends piece-wise affinely on the current system state. The very interesting approach is presented in Veselý, Rosinová, and Foltin, 2010. It creates a combination of infinite optimization horizon to ensure robust stability and finite time horizon for prediction of system outputs. The work leaved a lot of open problem in the areas of conservativeness reduction, better constraint handling and mainly the practical implementation problems.

To sum up the problem of robust stability in MPC still remains an open task. Both infinite time horizon and terminal constraint can be seen as a too restrictive in many cases. The explicit MPC reduces computational complexity but from the point of view of control theory does not bring any new ideas. The combination of terminal constraint to guarantee robust stability and explicit solution of MPC can be seen as a most advanced version of predictive controller which is currently available. However, the wide range of methods from the robust control theory indicates that there is an alternative approach possible.

2.1 Formulation of the work contribution

Based on the overview of existing methods and the previous summary of problems in MPC this thesis has the following goals:

- Create a design procedure of robust predictive controller for SISO systems with transfer function model with soft input constraints handling based on the variable gain approach which would be an alternative to GPC.
- Decrease conservatism of the robust predictive controller calculation in the state-space by replacing the quadratic stability with parameter dependent Lyapunov function.
- Solve the problems with practical implementation of the controller and prove applicability on the real plants.
- Improve the procedure of the off-line output feedback gain calculation.
- Incorporate the soft input constraints handling with guaranteed stability.

3 Stable Predictive Control for SISO systems

A new method of predictive controller design based on the transfer function model of the system is presented. It demonstrates that for SISO process it is possible to use a different design procedure to obtain simple robust predictive controller with guaranteed stability and also optimality of the output without the on-line optimization. Moreover, it can be augmented with a variable gain which behaves as a soft input constraint.

Consider the system model with polytopic uncertainty is $A_n(z^{-1})y(k) = B_n(z^{-1})\Delta u(k)$. Let the control algorithm for the $\Delta u(k)$ and the prediction of manipulated variable $\Delta u(k+i|k)$ be defined in the form:

$$\Delta u(k) = F_1(z^{-1})\Big(y(k) - w(k)\Big) + \sum_{j=1}^{N_y} k_j \Big(y(k+j|k) - w(k+j|k)\Big) \tag{1}$$

$$\Delta u(k+i|k) = F_2(z^{-1})\Big(y(k+i|k) - w(k+i|k)\Big) \tag{2}$$

Coefficients of the polynomials $F_1(z^{-1})$ and $F_2(z^{-1})$ are the controller parameters and values of k_j $(j = 1, \dots N_y)$ are parameters bounded with prediction of the output. Orders of $F_1(z^{-1})$, $F_2(z^{-1})$ and value of the prediction horizon N_y are tuning parameters. In order to guarantee the stability of the closed-loop system we must select $F_1(z^{-1})$ and $F_2(z^{-1})$ in a way that the characteristic polynomial has all poles inside the unit circle. Parameters k_j are obtained from the optimization which minimizes the cost function

$$J = \sum_{k=1}^{\infty} e(k)^2 \tag{3}$$

Where $e(k)$ is the control error which quantifies the difference between the set-point $r(k)$ and the process output $y(k)$.

Then we can use the method described in Hudzovič, 1967 to find the value of (3). This method calculates the sum of $e(k)^2$ from coefficients of the reduced polynomials of the numerator and denominator of the transfer function $E(z) = \frac{e(k)}{w(k+N_y|k)}$ with a step input signal $w(k+N_y|k)$. The obtained formula for the sum of $e(k)^2$ allows to find values of k_j parameters that minimizes J.

3.1 Input constraints

Consider that input $u(k) = u(k-1) + \Delta u(k)$ is bounded by $\pm U_{max}$. The input constraint can be modelled as a variable gain k_u:

$$k_u = \begin{cases} 1 & \text{if} \quad |u(k)| \leq U_{max} \\ \frac{U_{max}}{|u(k)|} & \text{if} \quad |u(k)| > U_{max} \end{cases} \tag{4}$$

Then the closed-loop system has structure as in fig. 1 and the

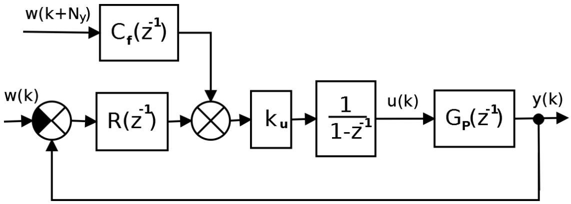

Figure 1: Closed-loop system with soft input constraints

characteristic polynomial $p(z^{-1})$ is:

$$p(z^{-1}) = \Big(A_n(z^{-1}) - F_1(z^{-1})B_n(z^{-1})k_u\Big)p_0(z^{-1}) \tag{5}$$

Characteristic polynomial $p(z^{-1})$ consists of two multiplied polynomials but variable gain k_u is only in the first polynomial.

$$p_1(z^{-1}) = \Big(A(z^{-1}) - F_1(z^{-1})B(z^{-1})k_u\Big) \tag{6}$$

From (6) and (4) we obtain one segment with two polynomials at its vertices.

$$p_a(z^{-1}) = \Big(A(z^{-1}) - F_1(z^{-1})B(z^{-1})k_{u_{min}}\Big) \tag{7}$$

$$p_b(z^{-1}) = \Big(A(z^{-1}) - F_1(z^{-1})B(z^{-1})1\Big) \tag{8}$$

There exist several theorems (Veselý and Harsányi, 2008) which solves the problem of the segment stability. It allows us to find k_{umin} (for example by several iterations with different values of minimal k_u). If we assure that the value of k_u needed to constrain input is always bigger than k_{umin} then the stability is guaranteed.

3.2 Example

Presented predictive control algorithm was tested on unstable magnetic levitation model CE 152. A polytopic model of the real system was created from transfer functions in three working points. We selected $F_1(z^{-1}) = F_2(z^{-1}) = F(z^{-1})$. We used the Edge theorem and the design procedure based on the D-curves. The following parameters of the controller guarantees the robust stability:

$$F(z^{-1}) = -21.7 + 41.66z^{-1} - 20z^{-2} \tag{9}$$

Parameters $k_1, \ldots, k_{N_Y}$ that minimizes (3) for $N_y = 10$ are:

$$\begin{aligned}\bar{k}^T =[&-4.5851 \quad 5.3933 \quad -0.1775 \quad -0.2352 \quad -0.0258 \\ &-0.1510 \quad 1.8977 \quad -10.4489 \quad 17.4912 \quad -9.2661]\end{aligned}$$

Value of the cost function is $J = 2.593$. Measured result on the real system compared with simulation is in fig. 2.

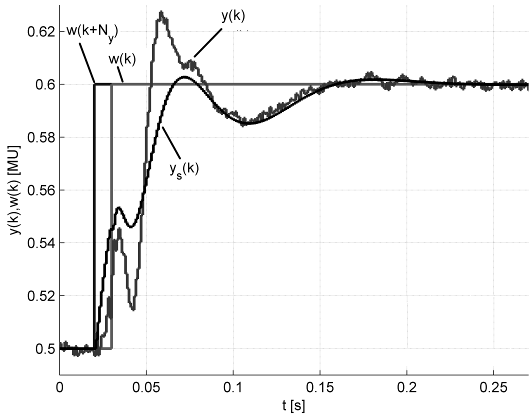

Figure 2: Time response of the ball position from simulation ($y_s(k)$) and from real process ($y(k)$)

To sum up, the presented method allows to design robustly stable predictive controller. Stability is guaranteed by finding adequate values of the controller parameters that moves poles of the characteristic polynomial into the stable area. In some cases well known methods of the classic PID design can be used to find these parameters. When the closed-loop system is stabilized, optimization is used to obtain remaining parameters and the closed-loop performance is improved. Selected cost function minimizes the sum of squares of errors.

4 Robust predictive control for MIMO systems

Let the polytopic linear discrete time system be described by

$$\tilde{x}(k+1) = \tilde{A}(\xi)\tilde{x}(k) + \tilde{B}(\xi)u(k), \qquad \tilde{y}(k) = \tilde{C}x(k) \tag{10}$$

The matrices $\tilde{A}(\xi)$ and $\tilde{B}(\xi)$ belong to the convex set Ω, with N vertices. System is augmented with integrator to force disturbance rejection and to achieve set-point tracking

$$z(k+1) = z(k) - \tilde{C}\tilde{x}(k) + w(k) \tag{11}$$

where $w(k)$ is a desired set-point value. Adding the integrator (11) to (10) one obtains:

$$x(k+1) = A(\xi)x(k) + B(\xi)u(k) + B_w w(k), \qquad y(k) = Cx(k) \tag{12}$$

$$\begin{aligned} x(k) &= \begin{bmatrix} \tilde{x}(k) \\ z(k) \end{bmatrix}, \quad A(\xi) = \begin{bmatrix} \tilde{A}(\xi) & 0 \\ -\tilde{C} & I \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ I \end{bmatrix}, \\ C &= \begin{bmatrix} \tilde{C} & 0 \\ 0 & I \end{bmatrix}, \quad B(\xi) = \begin{bmatrix} \tilde{B}(\xi) \\ 0 \end{bmatrix}, \quad y(k) = \begin{bmatrix} \tilde{y}(k) \\ z(k) \end{bmatrix} \end{aligned} \tag{13}$$

Optionally, the derivative part can be added:

$$y_d(k) = \tilde{y}(k-1) - \tilde{y}(k) = \tilde{C}\tilde{x}(k-1) - \tilde{C}\tilde{x}(k) \tag{14}$$

Then the system (12) is augmented as follows:

$$\begin{aligned} x(k) &= \begin{bmatrix} \tilde{x}(k) \\ z(k) \\ z_d(k) \end{bmatrix}, \ A(\xi) = \begin{bmatrix} \tilde{A}(\xi) & 0 & 0 \\ -\tilde{C} & I & 0 \\ I & 0 & 0 \end{bmatrix}, \ B_w = \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, \\ C &= \begin{bmatrix} \tilde{C} & 0 & 0 \\ 0 & I & 0 \\ -\tilde{C} & 0 & \tilde{C} \end{bmatrix}, \ B(\xi) = \begin{bmatrix} \tilde{B}(\xi) \\ 0 \\ 0 \end{bmatrix}, \ y(k) = \begin{bmatrix} \tilde{y}(k) \\ z(k) \\ y_d(k) \end{bmatrix} \end{aligned} \tag{15}$$

Simultaneously with (12) we consider the nominal model:

$$x(k+1) = A_0 x(k) + B_0 u(k) + B_w w(k), \qquad y(k) = Cx(k) \tag{16}$$

The nominal model (16) is used for the construction of the prediction model and (12) is considered as a real plant description providing the plant output. The prediction is carried out over a finite output horizon N_y and a control horizon N_u ($N_u \le N_y$). System augmented with prediction model is:

$$\begin{aligned} x_f(k+1) &= A_f x(k) + B_f u_f(k) + B_{wf} w_f(k) \\ y_f(k) &= C_f x_f(k) \end{aligned} \tag{17}$$

where

$$\begin{aligned} x_f(k) &= \begin{bmatrix} x(k) \\ \vdots \\ x(k+N_y) \end{bmatrix}, \quad w_f(k) = \begin{bmatrix} w(k) \\ \vdots \\ w(k+N_y) \end{bmatrix}, \\ u_f(k) &= \begin{bmatrix} u(k) \\ \vdots \\ u(k+N_y) \end{bmatrix}, \quad y_f(k) = \begin{bmatrix} y(k) \\ \vdots \\ y(k+N_y) \end{bmatrix} \end{aligned} \tag{18}$$

$$\begin{aligned} A_f &= \begin{bmatrix} A(\xi) \\ A_0 A(\xi) \\ \vdots \\ A_0^{N_y} A(\xi) \end{bmatrix}, \ C_f = \begin{bmatrix} C & 0 & \dots & 0 \\ 0 & C & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C \end{bmatrix}, \\ B_f &= \begin{bmatrix} B(\xi) & 0 & \dots & 0 \\ A_0 B(\xi) & B_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_0^{N_y} B(\xi) & A_0^{N_y-1} B_0 & \dots & B_0 \end{bmatrix}, \\ B_{wf} &= \begin{bmatrix} B_w & 0 & \dots & 0 \\ A_0 B_w & B_w & \dots & 0 \\ A_0^{N_y} B_w & A_0^{N_y-1} B_w & \dots & B_w \end{bmatrix} \end{aligned} \tag{19}$$

Matrices (19) in the system with prediction (17) are used only for calculation of robust controller gains (matrix A_f is augmented with zeros to square matrix). In the practical implementation $A(\xi)$ and $B(\xi)$ are replaced with A_0 and B_0.

There are two possible predictive control algorithms:

1. output feedback with proportional and integral part:

$$u_f(k) = Fy_f(k) - \bar{F}w_f(k) \tag{20}$$

2. controller in the simple feedback gain form

$$u_f(k) = Fy(k) \tag{21}$$

Matrices $F_{ij},\ i,j = 0,1,\ldots,N_y$ are output feedback gains with constant entries to be determined by minimizing the cost function

$$\begin{aligned} J &= \sum_{k=0}^{\infty} \tilde{J}(k) \\ \tilde{J}(k) &= \sum_{j=0}^{N_y} x^T(k+j)q_j x(k+j) + \sum_{j=0}^{N_u} u^T(k+j)r_j u(k+j) \\ &= x_f^T(k)Qx_f(k) + u_f^T(k)Ru_f(k) \end{aligned} \tag{22}$$

4.1 Output feedback

Controller feedback design uses the parameter dependent Lyapunov matrix $P(\xi) = \sum_{i=1}^{N} P_i\xi_i,\ P_i > 0$. The main idea is to linearise the non-linear terms in the matrix inequality to create an LMI which can be solved by some LMI solver. The LMIs are:

$$\begin{bmatrix} -P_i + Q & C_f^T F^T & (A_{fi} + B_{fi}FC_f)^T \\ FC_f & -R^{-1} & 0 \\ A_{fi} + B_{fi}FC_f & 0 & -P_i^{-1} \end{bmatrix} < 0 \tag{23}$$

$$\begin{aligned} &\begin{bmatrix} U_2 & C_f^T F^T & (H^T + B_{fi}FC_f)^T \\ FC_f & -R^{-1} & 0 \\ H^T + B_{fi}FC_f & 0 & -I \end{bmatrix} < 0 \\ &U_2 = -P_i + A_{fi}^T H^T + HA_{fi} - HH^T - \\ &\qquad - C_f^T F^T B_{fi}^T B_{fi}FC_f + Q \end{aligned} \tag{24}$$

This result was experimentally proved but the robust stability check must be added after the design of the controller gain F. The control algorithm is the guaranteed cost control with $J \leq J^* = V(k_0)$ where

$$J = \sum_{i=1}^{\infty} x_f^T(k)Qx_f(k) + u_f^T(k)Ru_f(k). \tag{25}$$

4.2 Practical implementation

Consider the feedback gain F of RMPC is calculated using the method described in the previous section. From (18) and (19) it is obvious that for the practical implementation the state observer and the model prediction without uncertainty must be created. At first in the prediction model only the nominal model is used. Then the matrices A_f and B_f have form:

$$A_{fp} = \begin{bmatrix} A(\xi_0) \\ A_0^2 \\ \vdots \\ A_0^{N_y+1} \end{bmatrix}, \quad B_{fp} = \begin{bmatrix} B(\xi_0) & 0 & \dots & 0 \\ A_0 B_0 & B_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_0^{N_y} B_0 & A_0^{N_y-1} B_0 & \dots & B_0 \end{bmatrix} \tag{26}$$

4.2.1 Realization 1

First is the realization where the predicted inputs and outputs are accessible and can be further used for constrains handling.

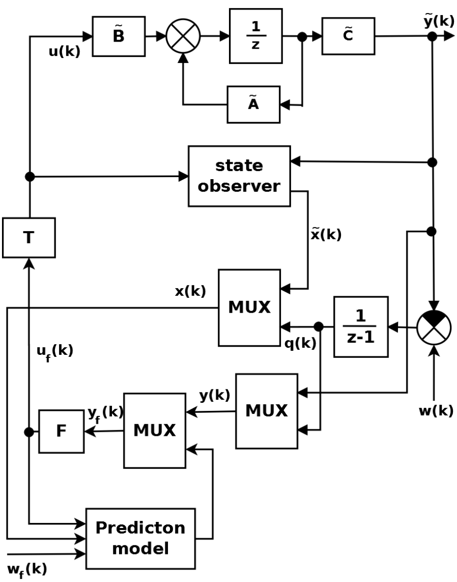

Figure 3: Closed-loop system configuration - realization 1

Matrix T select only first part $u(k)$ from vector $u_f(k)$ and has the form $T = \begin{bmatrix} I & 0 & \dots & 0 \end{bmatrix}$. The prediction model is

$$y_m(k) = C_m\Big(B_m u_f(k) + A_m x(k) + B_{mw} w_f(k)\Big) \tag{27}$$

$$y_f(k) = \begin{bmatrix} y(k) \\ y_m(k) \end{bmatrix}, \quad A_m = \begin{bmatrix} A_0^2 \\ \vdots \\ A_0^{N_y+1} \end{bmatrix}, \quad C_m = \begin{bmatrix} C & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C \end{bmatrix}$$

$$B_m = \begin{bmatrix} A_0 B_0 & B_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_0^{N_y} B_0 & A_0^{N_y-1} B_0 & \dots & B_0 \end{bmatrix}$$

4.2.2 Realization 2

Second realization is simplified and uses only necessary computing which is suitable for fast system with small sampling periods. The control algorithm is transformed to a simple set of feedback gains.

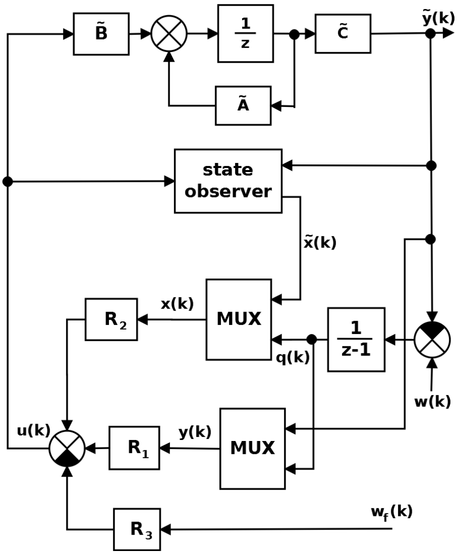

Figure 4: Closed-loop system configuration - realization 2

After some arrangement the control algorithm is:

$$u(k) = R_1 y(k) + R_2 x(k) + R_3 w_f(k) \tag{28}$$

4.3 New structure of feedback gain and prediction horizon

Consider the control algorithm is

$$u_f(k) = F y_f(k), \qquad F = diag(F_d, ..., F_d) \tag{29}$$

where dimension is $F_d \in R^{m \times 2l}$ and number of F_d blocks in F is $N_y + 1$. Diagonal structure reduces the computational complexity because it uses less variables for the LMI solving.

Now, we can define two prediction horizons. Let N_{yJ} be the prediction horizon used for calculation of feedback F_J which minimizes the cost function J. The block diagonal structure allows to realize controller with different prediction horizon N_{yF} simply by using more F_d blocks and create a matrix F with size $N_{yF} + 1$. Resulting controller is optimal for N_{yJ} steps prediction but for constraints handling has N_{yF} steps prediction where $N_{yJ} \leq N_{yF}$. Then the feedback gain and the controller is

$$u_f(k) = \begin{bmatrix} u(k) \\ \vdots \\ u(k+N_{yJ}) \\ \vdots \\ u(k+N_{yF}) \end{bmatrix} = \begin{bmatrix} F_d & 0 & \dots & 0 & 0 \\ 0 & \ddots & \dots & 0 & 0 \\ 0 & \dots & F_d & \dots & 0 \\ 0 & 0 & \dots & \ddots & 0 \\ 0 & 0 & 0 & \dots & F_d \end{bmatrix} y_f(k) \tag{30}$$

Note that the F_d highly depends on the selected horizon N_{yJ} and it is not enough to calculate F_d for zero horizon N_{yJ} and use it to create block diagonal matrix F.

4.3.1 Constraints - switched gain approach

The control algorithm with soft input constraints must on-line change the value of output feedback gain F. In the case of a multivariable system with ρ constraints the matrix F has the structure:

$$F = \gamma_1 F_1 + \gamma_{21} F_{21} + \gamma_{22} F_{22} + \ldots + \gamma_{2\rho} F_{2\rho} \tag{31}$$

where $\gamma_1, \gamma_{21}, \ldots, \gamma_{2\rho} \in \{0, 1\}$ and $\gamma_1 + \gamma_{21} + \ldots + \gamma_{2\rho} = 1$.

The constraint ρ can be either the single input constraint or a combination of more input constraints. The matrix F_1 is the output feedback for unconstrained case and matrix F_{2g} is the output feedback which guarantees constrained input for $g-th$ constraint. Usually F_1 is a faster controller with good performance but bigger values of input signal than matrices F_{2g}. The advantage over the approach with only two matrices F is that in a multivariable system only the dynamics of necessary input signals of the system can be changed if the constraint is reached.

The closed-loop system now contains a changing gain which makes it an LPV system. Robust stability must be checked by quadratic stability or the condition suitable for LPV system.

Parameters γ are filtered because fast step changes of the output feedback are not suitable in most cases. If the first order filter is used the γ parameters are

$$\begin{bmatrix} \gamma_1 \\ \gamma_{21} \\ \vdots \\ \gamma_{2\rho} \end{bmatrix} = \begin{bmatrix} \frac{bz^{-1}}{1-az^{-1}} \\ \vdots \\ \frac{bz^{-1}}{1-az^{-1}} \end{bmatrix} \begin{bmatrix} \gamma_1^\delta \\ \gamma_{21}^\delta \\ \vdots \\ \gamma_{2\rho}^\delta \end{bmatrix} \tag{32}$$

The only problem left is the algorithm for switching feedback gains that would guarantee a soft input constraints. We created an algorithm where the changes of γ_{2g}^δ, $g = 1, \ldots, \rho$ are based on the allowed zones in the range of the input signal. The zone where the input must be constrained is defined as $Z = \langle U_{min}, U_{min}+\epsilon) \cup (U_{max}-\epsilon, U_{max}\rangle$, where $\epsilon \in R$. For the system with two inputs the algorithm for γ_{2g}^δ is:

- $\gamma_1 = 1 - (\gamma_{21} + \gamma_{22})$
- $\gamma_{21}^\delta = 0$, $\gamma_{22}^\delta = 0$ if $u_1(k+h) \notin Z \wedge u_2(k+h) \notin Z$
- $\gamma_{21}^\delta = 1$, $\gamma_{22}^\delta = 0$ if $u_1(k+h) \in Z \wedge u_2(k+h) \notin Z$
- $\gamma_{21}^\delta = 0$, $\gamma_{22}^\delta = 1$ if $u_1(k+h) \notin Z \wedge u_2(k+h) \in Z$
- $\gamma_{21}^\delta = 0$, $\gamma_{22}^\delta = 0$ if $u_1(k+h) \in Z \wedge u_2(k+h) \in Z$

for all $h = 0, \ldots, N_y$.

4.3.2 Example

RMPC with block diagonal matrix F and the input constraints was tested on MIMO system with two inputs and two outputs. Output feedback in the unconstrained case with $N_y = 1$, $Q = I$, $R = diag\{r_1, r_2, r_1, r_2\}$, $r_1 = 18000$ and $r_2 = 53000$ calculated using LMI with linearisation is:

$$F_{d1} = \begin{bmatrix} -1.8797 & -0.2980 & 0.0509 & 0.0101 \\ -0.1988 & -1.7665 & 0.0093 & 0.0968 \end{bmatrix} \tag{33}$$

The input signal which behaves as a constrained one is simply created by the feedback with increased corresponding weight in the matrix R. Output feedback for constrained first input ($N_y = 1$, $Q = I$,

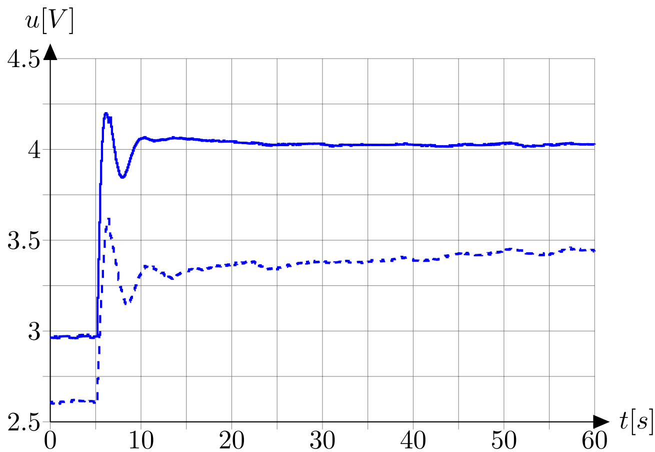

Figure 5: Measured system first input (solid line) and second input (dashed) with feedback gain $F = \gamma_1 F_1 + \gamma_{21} F_{21} + \gamma_{22} F_{22}$

$R = diag\{r_1, r_2, r_1, r_2\}$, $r_1 = 25000$ and $r_2 = 53000$) is:

$$F_{d21} = \begin{bmatrix} -0.9519 & -0.2348 & 0.0236 & 0.0090 \\ -0.2270 & -1.8908 & 0.0112 & 0.1052 \end{bmatrix} \tag{34}$$

Output feedback for constrained second input ($N_y = 1$, $Q = I$, $R = diag\{r_1, r_2, r_1, r_2\}$, $r_1 = 18000$ and $r_2 = 62000$) is:

$$F_{d22} = \begin{bmatrix} -1.7163 & -0.4987 & 0.0469 & 0.0163 \\ -0.2352 & -1.2028 & 0.0112 & 0.0608 \end{bmatrix} \tag{35}$$

Results are summarized in tab. 1 where is the value of settling time and overshoot for constrained and unconstrained case. Maximal value of input was set $U_{max} = 4.3$ and $\epsilon = 0.7$.

F	F_1	F_{22}	$\gamma_1 F_1 + \gamma_{21} F_{21} + \gamma_{22} F_{22}$
settling-time [s]	16	25	15
overshoot [%]	1	2	0.5
max. $u(k)$ [V]	4.5	4.25	4.25

Table 1: performance of the system output

The measured system input is in fig. 5. The result shows that

- the close-loop is robustly stable,
- maximal value of input U_{max} is not exceeded with new predictive controller algorithm and achieved performance is equal to the unconstrained controller.

5 Conclusion

This work was motivated by the recent results in robust predictive control. New prediction control algorithms allows to use methods from the robust control theory to design a predictive controller which has guaranteed robust stability and performance in the defined uncertainty set. Moreover, it shows that there is no problem to add the soft input constraints handling. This solves the problems of the on-line computational complexity, feasibility and stability problems in the standard formulation of MPC. Although, we can find different approach to RMPC in the literature some of the previous problem remains and not all of them are solved. The contributions and main results are summarized in the following section.

5.1 Contributions

Contributions are divided into two main topics. First is the new robust predictive controller for SISO system and second is about improvements in state-space formulation of the robust predictive controller.

5.1.1 Robust MPC for SISO systems

In the first part is presented a new controller algorithm based on the transfer function system model. Purpose of the development of this algorithm was the often seen application of GPC without constraints in the form of the RST controller. It is well known such a controller has no practical advantages over the classical linear one. However, there is still the lack of closed-loop stability and inability to control the unstable systems. Hence, we showed an alternative approach. It has some similar properties like GPC such as it is suitable for SISO systems and uses prediction of set-point. However, it guarantees the closed-loop stability and also possibly guaranteed robust stability. For the cost function optimization it uses unique method developed by Hudzovič, 1967 which calculates the sum of squares of errors from the provided discrete time transfer function. To the author's knowledge, this method wasn't used before for the controller design. Moreover, it was shown the input constraints handling can be added as a variable gain in the closed-loop. The closed-loop stability is still guaranteed. So, there is no reasonable need to use GPC without constraints because there exists an alternative solution with better properties. The results of this work were published in Vozák and Veselý, 2012; Vozák and Veselý, 2014a.

5.1.2 State-space robust predictive controller

This part was based on the previous works of Veselý, Rosinová, and Foltin, 2010; Nguyen, Veselý, and Rosinová, 2013. The following improvements were achieved.

Set-point and derivative term: In order to achieve the set-point tracking and zero steady-state error the integral term is added in the standard form. To improve the controller quality a new augmentation with derivative term was introduced in the predictive control. It augments the system model with new system output representing the first difference of the system output. The set-point extends the system and the prediction model with new matrix B_{wf} which is necessary for the correct simulation and the controller realization.

Reduced conservativeness: The original proposed version of the algorithm uses the quadratic stability for the output feedback gain calculation. Quadratic stability is considered to be rather conservative approach because it requires to find only one Lyapunov matrix P for the whole uncertainty. The natural way how to reduce conservativeness is to use the parameter dependent Lyapunov matrix. It uses N different Lyapunov matrices - one for each vertex of the polytopic model. The output feedback design was changed to the less conservative one which uses parameter dependent Lyapunov matrix and linearisation approach (Vozák and Veselý, 2014c).

Experimental testing of output feedback calculation method: The output feedback gain design method based on the parameter dependent Lyapunov function and linearisation does not have any formal proof of its stability. So, we needed to prove the quality of the presented output feedback calculation by random examples benchmark. It showed the method was very successful even in the case of high order unstable systems. The results were presented in Vozak and Vesely, 2013.

Alternative approach to output feedback calculation: We also tested a method which has performance defined as a required pole location in the circle in the complex plane with specified centre and radius (Vozák, 2014). The method comes with faster calculation of the feedback gain formulated as an LMI which does not need linearisation. The pole location approach makes this method more suitable for the state feedback design because we can't move poles to any location with the output feedback. So, it is a good alternative if there is a full state measurement available.

New constraints on matrix F elements: We introduced new LMI constraints for matrix F elements separated for proportional, integral and derivative part. The constraints defines the maximal and minimal value for every part. It helps to find suitable parameters of the controller in the cases when the designer needs to keep some properties due to some additional system attributes. For example lower derivative part due to noise amplification or limitation of the manipulated variable.

Practical implementation: The basic formulation of RMPC is represented as a state space model with one feedback gain. The practical implementation needed to add the set-point and separate the real system model from the prediction model and other augmentations. The result showed the state-observer or system states measurement must be added for correct predictions. We created two realization schemes (Vozák and Veselý, 2014b; Veselý and Vozák, 2014). First realization is minimal where the controller algorithm is reduced to a set of feedback gains. Second realization has accessible all predicted values of the system state, input and output. Both schemes are suitable for practical implementation and control of a real system.

Examples: A set of examples in this work shows each algorithm alternative to be working well not only on the simulation but also on the real systems control. The main purpose of the examples was to experimentally prove the proposed predictive controllers and the realization schemes can control real systems with no problems. This result is particularly important for the future applications.

New block diagonal structure: The last section introduces new structure of the feedback gain F. This special case of matrix F brings new possibilities for the prediction horizon and constraints handling. We can distinguish the horizon for the optimal controller calculation and the horizon for a real system control. This approach leads to reduced off-line computational complexity but still allows to have a necessarily long horizon for the prediction of system inputs and outputs. The direct influence of predictions on the manipulated variable through the matrix F is lost but the predictions are used for the change of feedback parameters depending on the system constraints. This approach is possible because the block diagonal structure preserve the correct predictions of system inputs and outputs. Moreover, we don't need to have only one feedback gain for the constrained case (as in the original proposed version) but we can have as many feedbacks as necessary for separated constraints of each input signal or a group of input signals.

5.2 Remarks and future research

Although, we solved the problems with stability, robust stability, online computational complexity and feasibility, of course, there is a price for this. The constraints needed to be changed from hard to the soft constraint and the off-line computational complexity increased noticeably. We consider there are still possibilities to find faster algorithms for output feedback design and reduce the time needed to find robust feedback. This is mostly important for the high order systems which is common situation in systems with long prediction horizon. Next interesting research area would be the switching algorithm for the change of the feedback gains depending on the constraints and predictions.

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