# Scientific council of Faculty of electrical engineering and information technology

**Slovak University of Technology in Bratislava**



**Ing. Quang Thuan Nguyen**


# ROBUST CONTROL OF DYNAMIC SYSTEMS

**Summary of doctoral dissertation**


A dissertation submitted for the degree of Philosophiae Doctor
in doctoral study programme: 9.2.7 Cybernetics

***Bratislava, June 2012***

This thesis was developed in a full-time doctoral study at Institute of Control and Industrial Informatics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava.

*Author*: **Ing. Quang Thuan Nguyen**
Institute of Control and Industrial Informatics
Faculty of Electrical Engineering & Information Technology
Slovak university of technology in Bratislava
Ilkovičova 3, 812 19 Bratislava

*Supervisor*: **Prof. Ing. Vojtech Veselý, DrSc.**
Institute of Control and Industrial Informatics
Faculty of Electrical Engineering & Information Technology
Slovak university of technology in Bratislava
Ilkovičova 3, 812 19 Bratislava

*Opponents*: ......................................................................................
......................................................................................
......................................................................................

......................................................................................
......................................................................................
......................................................................................

Thesis summary was submitted on ..........................................................

Date of PhD thesis defence ..................................................................
Committee for dissertation defence in the doctoral study programme is appointed by chairman of joint study programme committee on ........................

code of programme: 9.2.7, doctoral study programme: Cybernetics at Institute of Control and Industrial Informatics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Ilkovičova 3, 812 19 Bratislava.

*Chairman of joint study programme committee*:

......................................................................
Institute of Control and Industrial Informatics
Faculty of Electrical Engineering and Information Technology
Slovak University of Technology in Bratislava
Ilkovičova 3, 812 19 Bratislava

# Contents

# Nomenclature

## Mathematical denotations

$R^{m\times n}$ Matrix algebra of dimensions $m \times n$ with coefficients in set of real numbers

$I$ Identity matrix of corresponding dimensions

0 Matrix of corresponding dimensions with entries equal to 0

$A$ Matrix $A$, if not explicitly stated, is assumed to have compatible dimensions

$A^T$ Transpose of matrix A

$A^{-1}$ Inverse of matrix A

$\| A \|$ Norm of matrix A

$\lambda(A)$ Set of the eigenvalues of matrix A

$\lambda_{max}(A)$ The maximal real value of eigenvalue of matrix A

$A > 0$ Matrix $A$ is symmetric and positive definiteness

$A \geq 0$ Matrix $A$ is symmetric and positive semi-definiteness

$*$ A block that is transposed and complex conjugate to the respective symmetrically placed one

## Acronyms

BMI Bilinear Matrix Inequality

LMI Linear Matrix Inequality

PDQS Parameter Dependent Quadratic Stability

QS Quadratic Stability

MPC Model Predictive Control

NCS Networked Control System

LKF Lyapunov-Krasovskii Functional

# 1 Introduction

This submitted PhD thesis deals with problems of applying robust control theories to design robust output feedback controller for uncertain system in the fields of decentralized control, model predictive control, and networked control system.

## 1.1 Motivation

Conventional control theory with the requirement of exactly mathematical model of real process causes itself very hard to apply in the practice. Hence *robust control* theory is employed and developed to deal with system analysis and control design for such imperfectly known process models. Nowadays robust control becomes a highly effective method able to work in real conditions. Although a rich theory has been developed for the robust control of linear systems, but very little is known about the robust control of linear systems with *constraints*. Recently, this type of problem has been addressed in the context of *Model Predictive Control* (MPC). The success of MPC in industry is primarily due to the ease and the effect with which constraints on the inputs and states (outputs) can be included in the control problem formulation. However there is always a fundamental question about MPC, is its stability and robustness to model uncertainty.

Nowadays most industrial processes are naturally complex or *large-scale systems*. One of the main problems of complex large-scale system is highly structured. Hence, a *decentralized control* system employing distributed computation is essential. Another factor that significantly influences the problem of control system design for large-scale system is the inherent uncertainty in modeling the dynamic behavior of the system. The combination of uncertainties, high dimensions, and severe performance specifications creates a difficult control system design problem even were one to use a centralized control architecture. The need for robust decentralized control has been investigated for applications in complex and distributed systems, investigating robust stability of the interconnections of systems with local controllers.

Major advancements over the last decades of the 20th century in wired and wireless communication networks gave rise to the new paradigm of *Networked Control System* (NCS). This evolution of standalone control systems to NCSs brought many attractive advantages, which include low cost, simple installation and maintenance, increased system agility, higher reliability and greater flexibility. However the use of communication networks makes it necessary to deal with the effects of the network-induced imperfections and constraints, two among them are: *time-varying delays*, *packet dropouts*. With time varying *network-induced delay*, arbitrary *packet loss* and uncertainties of the controlled system, the insertion of the communication network in the feedback control loop makes the analysis and design of an NCS complex. Therefore, to handle network delays and packet loss in a closed-loop control system with model uncertainty over a communication network, advanced robust control methodologies are required.

The application of robust control theory in fields of decentralized control, model predictive control and robust networked control system is not really new, and there are many researches relating to these areas. However there are still many open questions which motivate us to research. In the next section, the open issues will be clearly identified based on review of existing works in the relevant areas.

## 1.2 Background

### 1.2.1 Robust control theory

Robust control theory of dynamical systems has started in the end of 70 years of the 20th century, when the foundations of optimal control of linear systems were established. The development of methods for robust control of linear dynamic systems and nonlinear dynamic systems described by state space equations with uncertainties especially was contributed by *Lyapunov stability theory*. *Quadratic stability (QS)* and *parameter dependent quadratic stability (PDQS)* criterions are used to analyze robust stability and design robust controllers for *polytopic linear model* in *time domain*. Description of uncertain systems using the convex polytope-type uncertainty has found its natural framework in the form of *Linear Matrix Inequality (LMI)* or *Bilinear Matrix Inequality (BMI)*. For convex polytopic uncertainty the Edge theorem and related works provide stability conditions for polytopic systems. It's well know that QS is conservative. To reduce QS' conservatism in analyzing robust stability of polytopic systems, PDQS has been introduced for both the continuous-time systems (Peaucelle et al. [2000], Rosinová et al. [2003]) and the discrete-time systems (Olivera et al. [1999]). Another important feature of stability condition is its applicability for a controller design (well known quality of this kind is dilation: it means that the system matrix does not appear in a product with unknown matrix-*convex problem*). Nonlinear convex inequality (*non-convex problem*) may be pre-converted into the LMI in some cases by different technics and algebraic implementation such as Schur complement, congruent transformation, Finsler lemma, Elimination lemma. In the paper Grman et al. [2005], authors provided survey of some recent robust stability conditions, their mutual comparison, and presents new robust parameter-dependent quadratical stability conditions for continuous-time and discrete-time systems with convex polytopic uncertainty.

In this dissertation, *parameter dependent quadratic stability (PDQS)* criterion is used to synthesise robust controller with *quadratic guaranteed cost* for the *polytopic model* of the uncertain system.

### 1.2.2 Decentralized control

The theory of complex or large-scale systems studies how relationships between subsystems give rise to the collective behaviors of a whole system interacts and forms relationship with its environments. Robustness is one of the attractive qualities of the decentralized control scheme, since such a control structure can be inherently resistant to a wide range of uncertainties both in subsystems and in the interconnections. Considerable effort has been made to consider robustness issues in the decentralized control structure and decentralized control design schemes, eg in Rosinová and Veselý [2006], Stankovič et al. [2007], and Zečevič and Šiljak [2004]. The above approaches compute decentralized control by a solution of the problem of the overall systems size. To reduce the problem size in decentralized control design for large scale systems, the diagonal dominance or block diagonal dominance concept can be adopted. Recently, the so called Equivalent Subsystems Method has been developed for decentralized control in frequency domain Kozaková et al. [2009]. The important point in this approach is that the controllers of equivalent subsystems can be independently tuned for stability according to specified stability and/or performance indices, so that the resulting decentralized controller guarantees the same stability/performance indices for the full system. The open question addressed by this thesis is how to design a robust decentralized controller based on subsystem approach in state space.

### 1.2.3 Model predictive control

MPC has been widely adopted in industry as an effective means to deal with multivariable constrained control problems. The success of MPC depends on the degree of precision of the plant model. In practice, modeling real plants inherently includes uncertainties that have to be considered in control design, that is the control design procedure has to guarantee stability, performance and robustness properties of closed-loop systems in the whole uncertainty domain. Robust-constrained MPC using linear matrix inequality (LMI) has been proposed by Kothare et al. [1996], where the polytopic and structured feedback uncertainty models have been used. The main idea of Kothare et al. [1996] is the use of infinite horizon control laws, which guarantees robust stability for the closed-loop system with state feedback. Constraints on control input are considered through invariant ellipsoids. The resulting control law in Kothare et al. [1996] is given by a state feedback gain matrix satisfying the respective LMI conditions for stability and additional LMI condition for constraints. Unconstrained robust output feedback MPC design with one-step ahead prediction is proposed in Veselý and Rosinová [2009]. The advantage of this method is that model prediction contains all the possible combinations of uncertainty. However, robust model prediction is just built for one-step-ahead prediction. In the paper Veselý et al. [2010], a construction of MPC for an uncertain polytopic system with constrained control based on model structure introduced in Veselý and Bars [2008] is proposed. The limitation of this approach is that the robust MPC used to design controller is built on the mix of the plant uncertain and nominal models; thus, it does not contain all the possible combinations of uncertainty in the original-plant polytopic model. As a result, the designed control law may not provide stability guarantee for the uncertain plant model and uncertain model prediction. This challenge will be investigated and solved in this thesis.

### 1.2.4 Networked control systems

#### Network-induced Delay

In the recent years, the stability analysis and controller synthesis for systems with time-delay are important in theory and practice. There are two approaches for controller design and study of closed-loop system stability in the time domain: Razumikhin theorem and Lyapunov-Krasovskii functional (LKF) approach. It is well known that the LKF approach often provides less conservative results than Razumikhin theorem (Friedman and Niculescu [2008]). To obtain necessary and sufficient condition for stability, it is necessary to use complete quadratic LKF as pointed out by Repin [1965]. By using the complete LKF approach, the least conservatism is obtained in comparison with above methods. However, to reduce the conservatism efficiently, two techniques have been developed. The first one is partitioning the delay to $N_d$ parts and using the discretized scheme of the Lyapunov-Krasovskii matrices (from the complete LKF) for these parts. It has been shown that if $N_d \to \infty$, the sufficient stability conditions for time delay systems approach to necessary ones (Gu et al. [2003]). From our review of literature, we know that, the guaranteed cost control approach has been extended to the uncertain time-delay systems, for the state feedback case, see (Yu and Chu [1999], Lee and Lee [1999]) and for output feedback (Chen et al. [2004], Xia et al. [2008]). However, in the above papers the obtained stability conditions are only sufficient, which can be far from necessary and sufficient ones, and the control state/output algorithm involves conservatism.

#### Packet Dropout

Two major approaches are usually used to accommodate the issue of packet loss in an NCS design. One way is that one first designs the control system without regard to the networks, and then determines a performance level that the networks should satisfy so that the closed-loop system maintains its performance (for example, stability) when some control and sensor signals are transmitted via the networks (Zhang et al. [2001]). The other approach is to treat the network protocol and traffic as given conditions and design the control strategies that explicitly take the network-induced issues into account (Azimi-Sadjadi [2003]; Xiong and Lam [2007]). In the last two decades, MPC has been widely adopted in industry and there are some researching results that have been presented in MPC for NCSs with packet-loss. Li et al. [2009] proposed a stabilizing MPC strategy for NCS with data packet loss between sensor and controller. Polytopic description was used to describe uncertainty of system. In Ding [2010b], the author proposed a MPC design method for NCS with double-sided packet loss. A packet-loss dependent Lyapunov function is used for stabilization, and the result is used for synthesizing model predictive control by parameterizing the infinite horizon control moves into a single state feedback law. One of the ways to overcome the resulting loss-packet problems is the use of prediction based compensation schemes (Grüne et al. [2009a]; Grüne et al. [2009b]). The open question addressed by this thesis is how to design a prediction based compensation schemes for NCS with arbitrary packet loss guaranteeing robustness (to be against model uncertainty), and insuring input constraints.

## 1.3 Objective of Thesis

The main objectives of the dissertation are these:

1. To design robust decentralized controller for large-scale system by using subsystem approach in state space.

2. To design robust output feedback model predictive control with input constraints to explicitly incorporate plant model uncertainty.

3. To employ novel algorithms with the least conservatism to design robust controller for uncertain NCS with time-varying network-induced delay.

4. To design robust predictive controller for uncertain NCS with arbitrary packet-loss.

## 1.4 Outline and Summary of Contributions

This dissertation is organized into five chapters including introduction and conclusion. The major contributions are covered by the following three chapters:

Chapter 2 provides an analogy of equivalent subsystems approach for decentralized control design in state space. The overall control problem is reduced to the subsystems size and therefore the proposed method excludes limit of system order in BMI solution; on subsystem level we adopt robust static output feedback control design with guaranteed cost, the interaction bound is considered via subsystem stability degree. The proposed design method is based on the Generalized Gershgorin Theorem and V-K iteration procedure in LMI to check the robustness and performance of complex system.

Chapter 3 develops a new synthesis method to design a robust output feedback MPC with input constraints to extend model predictive control in papers Veselý and Rosinová [2009] and Veselý et al. [2010]. The proposed predictive control strategy expands the predictive control algorithm in paper Veselý and Rosinová [2009] to longer prediction horizon and control horizon. Additionally, an integrator is added to the controller design procedure to reject disturbances and maintain the process at the optimal operating conditions or setpoints. Two input constraints approaches such as heuristic one and invariant set are concerned. The main contribution is that all the timedemanding computations of the output feedback gain matrices are realized off-line.

Chapter 4 includes two parts which introduce novel methods to design robust output feedback controller for NCSs with the time-varying delay and packet-loss. The first presents two new approaches such as complete LKF and discretized LKF to design robust output feedback PID controllers achieving a guaranteed cost such that the NCSs can be stabilized for all admissible polytopic-type uncertainties and time-varying delays with less conservatism than previous works. In the second part, a robust output feedback linear model predictive control scheme over a network with double-sided packet loss is implemented. The main idea is based on the combination of compensation mechanism and robust model predictive control design approach in Chapter 3. As a result, networked predictive control systems with loss packet are modeled as switched linear systems. This enables us to apply the theory of switched systems to establish the stability condition of networked model predictive control.

The evaluation, summarization of the dissertation and discuss open problems as well as broadens our visions with some future works come in the last chapter 5.

# 2 Robust decentralized controller design

Consider the following linear large-scale continuous system including $M$ subsystems with polytopic uncertainty described

$$\dot{x}(t) = A(\xi)x(t) + B(\xi)u(t), \quad y(t) = Cx(t) \tag{2.1}$$

where matrices $A(\xi), B(\xi)$ belong to a convex and bounded polytope $S$:

$$S := \left\{ \; A(\xi) = \sum_{k=1}^{N} \xi_k A_k, B(\xi) = \sum_{k=1}^{N} \xi_i B_k, \sum_{k=1}^{N} \xi_k = 1, \xi_k \geq 0 \; \right\} \tag{2.2}$$

Matrices $A_k, B_k$ can be split in two parts

$$A_k = A_{dk} + A_{mk} \; ; \; B_k = B_{dk} + B_{mk} \; ; \; k = \{1, 2...N\} \tag{2.3}$$

where $A_{dk} = diag\{A_{jj}^k\}, B_{dk} = diag\{B_{jj}^k\}, j = \{1, ..., M\}$ are block diagonal matrices of the corresponding $j - th$ subsystems and matrices $A_{mk} = A_k - A_{dk}, B_{mk} = B_k - B_{dk}$ which are diagonal off matrices, which describe the interactions between remain $M - 1$ subsystems and $j - th$ subsystem. The output matrix $C = diag\{C_j\}, j = \{1, ..., M\}$.

We study the problem to find the decentralized stabilizing PI static output feedback controller for the overall system, described by control law

$$u_j(t) = K_{Pj} y_j(t) + K_{Ij} \int y_j(t)dt = K_{Pj} C_j x_j(t) + K_{Ij} z_j(t) \tag{2.4}$$

based on local robust controllers design, so that only problems of subsystems dimension have to be solved.

With control algorithm (2.4), closed-loop feedback of the overall system is obtained as follows

$$\dot{x}(t) = \left( \; A_{dc}(\xi) + A_{mc}(\xi) \; \right) x(t) \tag{2.5}$$

where $i, j = \{1, ..., M\}$; $A_{dc}(\xi) = \sum_{k=1}^{N} \xi_k diag\{A_{cjj}^k\}, \;\; A_{cjj}^k = A_{jj}^k + B_{jj}^k F_j C_j$; $A_{mc}(\xi) = \sum_{k=1}^{N} \xi_k \{A_{cij}^k\}$, $A_{cij}^k = A_{ij}^k + B_{ij}^k F_j C_j \; (j \neq i), A_{cij}^k = 0(j = i)$.

Simultaneously, with the system (2.5) we consider the following auxiliary complex system

$$\dot{x}(t) = \left( \; G_d(\xi) + G_m(\xi) \; \right) x(t) \tag{2.6}$$

where $i, j = \{1, ..., M\}$;$G_d(\xi) = \sum_{k=1}^{N} \xi_k diag\{-\gamma_{jj}^k\}, \gamma_{jj}^k > 0$;$G_m(\xi) = \sum_{k=1}^{N} \xi_k \{\rho_{ij}^k\}$ , $\rho_{ij}^k > 0(j \neq i), \rho_{ij}^k = 0(j = i)$.

Due to Gershgorin theorem the stability of system (2.6) on $k$-vertex is guaranteed if

$$\gamma_{jj}^{k} \geq \sum_{i=1;i\neq j}^{M} \rho_{ij}^{k}, j=\{1,2,...,M\}, k=\{1,2,...,M\}. \tag{2.7}$$

**Assertion 2.1** *Let $\gamma_{jj}^{k}$ to be stability degree of j-subsystem for k-vertex and $\rho_{ij}^{k} = \| A_{ij}^{k} + B_{ij}^{k} F_j C_j \|$;i,j=$\{1,2,...,M\}$, if for the system (2.5), the condition (2.7) holds,the system is stable in $k-$vertex, $k=\{1,2,...,N\}$.*

**Controller design procedure**

From above, the following steps may give positive results to robustly stabilize the closed-loop large-scale system.

1. Design the robust controller with gain matrix $F_j$ for $j$-subsystem such a way that (2.7) holds and the following subsystem is stable

$$A_{cjj}^{k} + \gamma_{jj}^{k} I, j=\{1,2,...,M\}, k=\{1,2,...,N\} \tag{2.8}$$

2. Design a gain matrix $F_j$ so that the following condition holds

$$\begin{bmatrix} (\rho_{ij}^{k})^2 I & (A_{cij}^{k})^T \\ A_{cij}^{k} & I_{ij} \end{bmatrix} \geq 0; i,j=\{1,2,...,M\}; i\neq j; k=\{1,2,...,N\} \tag{2.9}$$

3. Design a gain matrix $F_j$ so that $trace(D_j^k)$ is minimized and where $D_j^k$ $= diag\{\rho_{ij}^{k}\}$, $i \neq j$

4. When all subsystems are robust stable with guaranteed cost, check the robust stability of complex system.

5. If the complex system is not robustly stable with performance increase stability degree $\gamma_{jj}^{k}$ and return to first point.

6. If the complex system is not robustly stable with performance, an alternative way to get robust stability of complex system is as follows: put $F=\{F_j\}=\alpha F; \alpha > 0$ and using V-K iteration procedure using LMI for $\alpha=1$ and complex system calculate matrices $P_k, k=\{1,2,...N\}; G, H$ and then $\alpha > 0$

We have to note that above procedure do not guarantee the stability of complex system, but above procedure gives the way how we can obtain the robust stability of complex system.

# 3 Robust model predictive control design

## 3.1 Robust MPC

Let the polytopic model of the plant to be controlled be described by the following linear discrete time difference equation

$$x(t+1) = A(\xi)x(t) + B(\xi)u(t), \quad y(t) = Cx(t) \tag{3.1}$$

where matrices $A(\xi), B(\xi)$ belong to a convex and bounded polytope $S$ (2.2).

Consider the following integrator to successfully force disturbance rejection and setpoint tracking

$$z(t+1) = z(t) + e(t) = z(t) + Cx(t) - w(t) \tag{3.2}$$

where $w(t)$ is desired reference.

After adding integrator (3.2) into process model (3.1), we assume that the model (without change of denotation) in the following compact form:

$$x(t+1) = A(\xi)x(t) + B(\xi)u(t) + B_w w(t) \tag{3.3}$$

where $x(t) := \left[x^T(t) \quad z^T(t)\right]^T$; $B_w^T = -[0 \quad I]$; $B_w(\xi)^T = [B(\xi) \quad 0]$ and $A(\xi) := [A(\xi) \quad 0\ ;\ C \quad I]$.

Consider the output feedback predictive control algorithm with a prediction and control horizon $N_y, N_u$ $(N_u \leq N_y)$ for system (3.3) as follows

$$u(t+i) = \begin{cases} \sum_{j=0}^{N_y} \left[F_{ij}Cx(t+j) - F_{ij}C_w w(t+j)\right], \ 0 \leq i < N_u \\ 0, \ i = \{N_u, ..., N_y - 1\} \end{cases} \tag{3.4}$$

where $C := diag\{C, I\}, C_w^T = [I \quad 0]$. Input constraints are assumed to be

$$\|u_i(t+k|t)\| \leq \overline{u}_i; i = \{1, 2, ..., m\}, k = \{0, 1, ..., N_u - 1\} \tag{3.5}$$

where $\overline{u}_i$ is the maximum value of the $i - th$ input control $u_i(.)$.

The states of the system (3.3) for the instant $t + k$, $k = \{0, ..., N_y - 1\}$

$$x(t+k+1) = A(\xi)x(t+k) + B(\xi)u(t+k) + B_w w(t+k) \tag{3.6}$$

Considering $x_f(t) = [x^T(t) \ ... \ x^T(t+N_y-1|t)]^T$; $\varpi(t) = [w^T(t) \ ... \ w^T(t+N_y|t)]^T$; and $\nu(t) = [u^T(t) \ u^T(t+1) \ ... \ u^T(t+N_u-1)]^T$, state model prediction is obtained as follows

$$A_f(\xi)x_f(t+1) = A_x(\xi)x(t) + B_f(\xi)\nu(t) + B_{wf}\varpi(t) \tag{3.7}$$

where $B_{wf} = diag\{B_w\} \in R^{nN_y \times lN_y}$; $B_f(\xi) = \left[B_d^T(\xi) \quad Z^T\right]^T \in R^{nN_y \times mN_u}$, $B_d(\xi) = diag\left\{B(\xi)\right\} \in R^{nN_u \times mN_u}$, $Z = \{0\} \in R^{n(N_y - N_u) \times mN_u}$;

$$A_f(\xi) = \begin{bmatrix} I & 0 & 0 & \cdots & 0 & 0 \\ -A(\xi) & I & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -A(\xi) & I \end{bmatrix}; \; A_x(\xi) = \begin{bmatrix} A(\xi) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Using definitions of $x_f(t)$ and $\varpi(t)$, the predictive control algorithm (3.4) is also obtained in the following form

$$\nu(t) \quad = F_x C x(t) + F_f C_f x_f(t+1) - F\varpi(t) = F C_m \eta(t) - F\varpi(t) \qquad (3.8)$$

where $\eta^T(t) = [x^T(t) \; x_f^T(t+1)]^T, C_f = diag\{C, ..., C\} \in R^{lN_y \times nN_y}, C_m = diag\{C, C_f\}$ and $F_x = \{F_{i0}\}$, $F_f = \{F_{ij}\}_{j>0}$, $F = [F_x \quad F_f]$.

Substituting $\nu(t)$ in the form of (3.8) into the model prediction (3.7), the closed-loop model prediction is obtained as follows

$$A_{cf}(\xi) x_f(t+1) = A_{cx}(\xi) x(t) - B_{cf}(\xi)\varpi(t) \qquad (3.9)$$

where $A_{cf}(\xi) = A_f(\xi) - B_f(\xi) F_f C_f$, $A_{cx}(\xi) = A_x(\xi) + B_f(\xi) F_x C$, $B_{cf}(\xi) = B_f(\xi) F + B_{wf}$.

The problem now is to design a robust MPC with output feedback (3.4) for a given $N_y$, $N_u$ which guarantees the closed-loop system (3.9) stability (PDQS), robustness and guaranteed cost for the following cost function(over the infinite optimization horizon):

$$J = \sum_{t=0}^{\infty} J(t) = \sum_{t=0}^{\infty} \left[\eta^T(t) Q \eta(t) + v(t)^T R v(t)\right] \qquad (3.10)$$

where $Q = diag\{Q_0, ..., Q_{N_y}\}$ and $R = \{R_0, ..., R_{(N_u-1)}\}$, $(Q_k = q_k I, R_k = r_k I; q_k \geq 0, r_k > 0)$.

Main result on robust MPC design can be summarized in the following theorem.

**Theorem 3.1** *The closed loop system (3.9) is robustly stable with guaranteed cost $J_0$ and parameter dependent Lyapunov function if and only if there exist matrices $H \in R^{nN_y \times n(N_y+1)}$, $P(\xi) = P^T(\xi) > 0$ and gain matrix $F$ such that the following bilinear matrix inequality holds*

$$W(\xi) = D(\xi) + A_m^T(\xi) H + H^T A_m(\xi) + Q + C_m^T F^T R F C_m \leq 0 \qquad (3.11)$$

*where $D(\xi) \in R^{n(N_y+1) \times n(N_y+1)}$, $A_m(\xi) = [A_{cx}(\xi) \quad -A_{cf}(\xi)]$, $D(\xi) = diag\{-P(\xi), 0, ..., 0, P(\xi)\}$.*

In conclusion, for the initial condition $x(t_0) = x_0$, the Robust MPC law is summarized by the following algorithm:

1. Off-line compute gain matrix $F$ from solution of the optimization problem (3.11).
2. Get new value of current outputs from plant and predicted outputs from the model prediction.
3. Compute $v(t)$or $u(t+i), i = 0, 1, ...N_u - 1$.
4. Apply $u(t)$ to the plant and $v(t)$ to the model prediction.
5. $t := t + 1$. Go to 2.

## 3.2 Robust MPC with input constraints

### Heuristic method

For the obtained value of $u(t)$ in the previous section, control algorithm with input constraints is constructed as the follows:

$$u_c(t) = k_u u(t) \tag{3.12}$$

where $k_u$ is defined as the follows

$$k_u = \begin{cases} 1 & \text{if} \quad \|u(t)\| \leq \|u_M\| \\ \frac{\|u_M\|}{\|u(t)\|} & \text{if} \quad \|u(t)\| > \|u_M\| \end{cases} \tag{3.13}$$

The equation (3.13) implies $0 < k_u \leq 1$. For a given positive number $k_{u\min} > 0$, suppose $k_u$ satisfies

$$k_{u\min} \leq k_u \leq 1 \tag{3.14}$$

Substituting $u_c(t)$ instead of $u(t)$in (3.4) one obtains the closed-loop system (3.9). When the gain matrix $F_o$ is known for $k_u F_o$ we obtain a new closed loop system

$$x_f(t+1) = A_c(\xi, k_u)\, x(t) \tag{3.15}$$

where $k_u$ plays a role of new bounded uncertainty defined by (3.14). For this case, the number of vertices increases to $2N$ putting $k_u = k_{u\min}, k_u = 1$ and a problem is to find such value of $k_{u\min}$ that guarantees the closed-loop robust stability with performance. The following Lemma is used to check robust stability for the closed-loop system (3.15).

**Lemma 3.1** *Under the same conditions as in Theorem 1 if for a given $k_u = k_{u\min}$ and $k_u = 1$ the closed-loop system (3.15) is*

1. *quadratically stable with guaranteed cost if there exists feasible solution of (3.11) with respect to matrices* $P_i = P_j = P = P^T > 0$ *and* $H$ *for* $i = \{1, 2, ...2N\}$*;*

2. *parameter dependent quadratically stable with guaranteed cost if there exists feasible solution of (3.11) with respect to matrices* $P_i > 0$ *and* $H$ *for* $i = \{1, 2, ...2N\}$*;*

Value of $k_{u\,\min}$ is chosen as small as possible until the robust stability of the closed loop system (3.15) is also guaranteed, respectively the condition (3.11) holds.

**Invariant set method**

To derive sufficient stability conditions for input constraints for (3.9), we consider that the positive invariant region (Rohal-Ilkiv [2004]), with respect to closed-loop system motion can be defined by the ellipsoidal Lyapunov function set given as follows

$$\Omega(P(\xi)) = \{\eta(t) \in R^{n(N_y+1)} : \eta^T(t)\widehat{P}(\xi)\eta(t) \leq \theta\} \tag{3.16}$$

where $\widehat{P}(\xi) = diag\{P(\xi), ..., P(\xi)\} \in R^{n(N_y+1)\times n(N_y+1)}$ and $\theta$ is a positive real parameter which determines the size of $\Omega(P(\xi))$.

Consider $D_{i_d}F$ denotes the $i_d - th$ row of matrix $F$ where $D_{i_d} = [0...0\;1\;0...0] \in R^{l\times mN_u}$ and define

$$\text{Ł}(F) = \left\{ \begin{array}{r} \eta(t) \in R^{n(N_y+1)} : \|D_{i_d}FC_m\eta(t)\| \leq \bar{u}_i; id = i + (j-1)m; \\ i = \{1, ..., m\}; j = \{1, ..., N_u\} \end{array} \right\} \tag{3.17}$$

The condition of input constraints reduces to LMI given by the following theorem (Veselý et al. [2010])

**Theorem 3.2** *The inclusion* $\Omega(P(\xi)) \subseteq L(F)$ *is for output feedback control equivalent to*

$$\begin{bmatrix} \widehat{P}(\xi) & * \\ D_{i_d}FC_m & \lambda_{i_d} \end{bmatrix} \geq 0 \tag{3.18}$$

*for all* $i_d = i + (j-1)m; i = \{1, ..., m\}; j = \{1, ..., N_u\}$*, where* $\lambda_{i_d} \in < 0, \frac{\overline{u}_i^2}{\theta} >$.

# 4 Robust networked control systems design

## 4.1 Robust controller design for NCS with time-varying network-induced delay

Consider the following linear time-delay system

$$\begin{aligned} &\dot{x}(t) = A(\xi)x(t) + A_d(\xi)x(t-\tau(t)) + B(\xi)u(t) \\ &y(t) = Cx(t) \\ &x(t) = \varphi(t), t \in [-\tau_M; 0] \end{aligned} \tag{4.1}$$

We assume that a real-time communication network is integrated into feedback control loops of system (4.1), and the network induced delay in NCS $\tau(t)$ is given by $0 < \tau(t) \leq \tau_M$ and the derivative of $\tau(t)$ is bounded by $|\dot{\tau}(t)| \leq \mu \leq 1$.

Consider the following PID control algorithm for system (4.1)

$$u(t) = K_P y(t-\tau) + K_I \int_0^t y(t-\tau)dt + K_D \frac{d}{dt} y(t-\tau) \tag{4.2}$$

Using Newton-Leibniz formulas, the extended closed-loop system of system (4.1) with PID control algorithm (4.2) is obtained as follows:

$$M_d(\xi)\dot{X}(t) + A_c(\xi)X(t) + A_{dc}(\xi)\int_{t-\tau}^t \dot{X}(s)ds + A_{dd}(\xi)\int_{t-\tau}^t \ddot{X}(s)ds = 0 \tag{4.3}$$

where $X(t) = \begin{bmatrix} x^T(t) & z^T(t) \end{bmatrix}^T$, $z(t) = \int y(t-\tau)dt$.

Given positive definite symmetric matrices Q, R and S, we will consider the cost function

$$J = \int_0^\infty J(t)dt \tag{4.4}$$

where $J(t) = X^T(t)QX(t) + u^T(t)Ru(t) + \dot{X}^T(t-\tau)S\dot{X}(t-\tau)$.

### Complete quadratic Lyapunov-Krasovskii Functional approach

**Theorem 4.1** *Consider the uncertain linear time-delay system (4.1) with network-induced delay $\tau$ satisfying $0 < \tau \leq \tau_M, \dot{\tau} \leq \mu \leq 1$ and the cost function (4.4). If there exist a PID controller of form (4.2), scalar $J_0$, and matrices $P_i > 0$, $G_i > 0$, $G_{1i} > 0$, $G_{2i} > 0$, $G_{3i} > 0$ $(i = 1, ..., N)$, $N_1$, $N_2$, $N_3$, $N_4$, and $N_5$ that satisfy the following matrix inequality*

$$W_i = \begin{bmatrix} w_i^{11} & w_i^{12} & w_i^{13} & w_i^{14} & w_i^{14} \\ * & w_i^{22} & w_i^{23} & w_i^{24} & w_i^{25} \\ * & * & w_i^{33} & w_i^{34} & w_i^{35} \\ * & * & * & w_i^{44} & w_i^{45} \\ * & * & * & * & w_i^{55} \end{bmatrix} \leq 0 \tag{4.5}$$

*where*

$$
\begin{aligned}
w_i^{11} &= N_1 M_{di} + M_{di}^T N_1^T + \tau_M G_{1i} + \mu G_{3i} + C_D^T F_D^T R F_D C_D + S \\
w_i^{12} &= N_1 A_{ci} + M_{di}^T N_2^T + P_i + C_D^T F_D^T R F C_n \\
w_i^{13} &= N_1 A_{dci} + M_{di}^T N_3^T - C_D^T F_D^T R F_P C_P \\
w_i^{14} &= M_{di}^T N_4^T \\
w_i^{15} &= N_1 A_{ddi} + M_{di}^T N_5^T + (1-\mu)\mu G_{3i} - C_D^T F_D^T R F_D C_D - S \\
w_i^{22} &= N_2 A_{ci} + A_{ci}^T N_2^T + \mu G_i + C_n^T F^T R F C_n + Q \\
w_i^{23} &= N_2 A_{dci} + A_{ci}^T N_3^T + (1-\mu) G_i + G_{2i} - C_n^T F^T R F_P C_P \\
w_i^{24} &= A_{ci}^T N_4^T + G_{2i} \\
w_i^{25} &= N_2 A_{ddi} + A_{ci}^T N_5^T - C_n^T F^T R F_D C_D \\
w_i^{33} &= N_3 A_{dci} + A_{dci}^T N_3^T - (1-\mu) G_i - \tfrac{1}{\tau_M} G_{1i} - G_{2i} + C_P^T F_P^T R F_P C_P \\
w_i^{34} &= A_{dci}^T N_4^T - G_{2i} \\
w_i^{35} &= N_3 A_{ddi} + A_{dci}^T N_5^T + C_P^T F_P^T R F_D C_D \\
w_i^{44} &= -G_{2i} - \tfrac{1}{\tau_M} G_{1i} \\
w_i^{45} &= N_4 A_{ddi} \\
w_i^{55} &= N_5 A_{ddi} + A_{ddi}^T N_5^T - (1-\mu) G_{3i} + C_D^T F_D^T R F_D C_D + S
\end{aligned}
$$

*Then the uncertain system (4.1) with controller (4.2) is parameter-dependent quadratically- asymptotically stable and the cost function (4.4) satisfies the following bound*

$$J \le J_0 = \sqrt{\lambda_{MP}^2 + \lambda_{MG}^2 + \lambda_{MG1}^2 + \lambda_{MG2}^2 + \lambda_{MG3}^2} * J_M \tag{4.6}$$

*where* $\lambda_{MP} = \max_{i=1..N}(\lambda_{max}(P_i))$, $\lambda_{MG} = \max_{i=1..N}(\lambda_{max}(G_i))$, $\lambda_{MG1} = \max_{i=1..N}(\lambda_{max}(G_{1i}))$, $\lambda_{MG2} = \max_{i=1..N}(\lambda_{max}(G_{2i}))$, $\lambda_{MG3} = \max_{i=1..N}(\lambda_{max}(G_{3i}))$,

$$J_M = \sqrt{\begin{array}{l} \|x_0\|^4 + \left(\int_{-\tau}^0 \|\varphi(s)\|^2 ds\right)^2 + \left(\int_{-\tau}^0 d\theta \int_\theta^0 \|\dot\varphi(s)\|^2 ds\right)^2 \\ + \left(\int_{-\tau_M}^{-\tau} \|\varphi(s)\|^2 ds\right)^2 + \left(\int_{-\tau}^0 \|\dot\varphi(s)\|^2 ds\right)^2 \end{array}}$$

**Discretized Lyapunov-Krasovskii Functional approach**

Let us now to suppose that the time interval $[t-\tau(t), t]$ is partitioned into $N_d$ parts. The discretization-like method is employed considering the state vector shifted by a fraction $\frac{\tau(t)}{N_d}$ of the delay. Based on partitioning scheme of time-varying delay and using IQC (Ariba and Gouaisbaut [2007]), a new discretized Lyapunov-Krasovskii functional method is obtained to design a PI controller achieving a guaranteed cost such that the NCSs can be stabilized for all admissible uncertainties and time-varying delays with the least conservatism in the following theorem

**Theorem 4.2** *Consider the uncertain linear time-delay system (4.1) with network-induced delay $\tau(t)$ satisfying $0 < \tau(t) \leq \tau_M, \|\dot{\tau}(t)\| \leq \mu \leq 1$ and the cost function (4.4, $S = 0$). Assume that there exists a PI controller of form (4.2), scalar $J_0$, and matrices $P_i > 0, Q_{0i} > 0, Q_{1i} > 0, Q_{2i} > 0, R_{0i} > 0, R_{1i} > 0 (i = \{1, \ldots, N\}), N_1, N_2, N_3$ that satisfy the following matrix inequality*

$$
\begin{aligned}
W_i &= \begin{bmatrix} w_{11}^i & w_{12}^i & w_{13}^i \\ * & w_{22}^i & w_{23}^i \\ * & * & w_{33}^i \end{bmatrix} + M_{Q_0}^T \begin{bmatrix} \mu Q_{0i} & (1-\mu)Q_{0i} \\ * & -(1-\mu)Q_{0i} \end{bmatrix} M_{Q_0} \\
&+ M_{R_0}^T \begin{bmatrix} \tau_M R_{0i} & 0 \\ * & -\frac{1}{\tau_M} R_{0i} \end{bmatrix} M_{R_0} + M_{R_1}^T \begin{bmatrix} \frac{\tau_M}{N_d} R_{1i} & 0 \\ * & -\frac{N_d}{\tau_M} R_{1i} \end{bmatrix} M_{R_1} \\
&+ \left\{ M_{Q_{1a}}^T Q_{1i} M_{Q_{1a}} - \left(1 - \frac{\mu}{N_d}\right) M_{Q_{1b}}^T Q_{1i} M_{Q_{1b}} \right\} \\
&\leq 0
\end{aligned}
\tag{4.7}
$$

*where*

$$
\begin{aligned}
&w_{11}^i = N_1 + N_1^T + \left[\tfrac{\tau_M \mu (N_d - 1)}{2N_d}\right]^2 \tfrac{1}{1-\mu} Q_{2i}, \; w_{12}^i = N_1 A_{ci} + N_2^T + P_i \\
&w_{13}^i = N_1 A_{dci} I_P + N_3^T \\
&w_{22}^i = N_2 A_{ci} + A_{ci}^T N_2^T + C_n^T F^T R F C_n + Q \\
&w_{23}^i = N_2 A_{dci} I_P + A_{ci}^T N_3^T - C_n^T F^T R F_P C_P I_P \\
&w_{33}^i = N_3 A_{dci} I_P + I_P^T A_{dci}^T N_3^T - diag\{0_{N_d n}, Q_{2i}\} + I_P^T C_P^T F_P^T R F_P C_P I_P
\end{aligned}
$$

*matrices $M_{Q0}$, $M_{R0}$, $M_{R1} \in R^{2n \times (N_d+3)n}$ and matrices $M_{Q_{1a}}, M_{Q_{1b}} \in R^{N_d n \times (N_d+3)n}$ (see Dissertation for more detail), then the uncertain system (4.1) with controller (4.2) is parameter-dependent quadratically asymptotically stable and the cost function (4.4) satisfies the following bound*

$$
J \leq J_0 = \sqrt{\lambda_{MP}^2 + \lambda_{MQ_0}^2 + \lambda_{MR_0}^2 + \lambda_{MR_1}^2 + \lambda_{MQ_1}^2} * J_M \tag{4.8}
$$

*where* $\lambda_{MP} = \max\limits_{i=\{1,\ldots,N\}} (\lambda_{max}(P_i))$, $\lambda_{MQ_0} = \max\limits_{i=1\ldots N} (\lambda_{max}(Q_{0i}))$, $\lambda_{MR_0} = \max\limits_{i=1\ldots N} (\lambda_{max}(R_{0i}))$, $\lambda_{MR_1} = \max\limits_{i=1\ldots N} (\lambda_{max}(R_{1i}))$, $\lambda_{MQ_1} = \max\limits_{i=1\ldots N} (\lambda_{max}(Q_{1i})))$,

$$
J_M = \sqrt{\begin{aligned} &\|x_0\|^4 + \left(\int\limits_{-\tau_M}^{0} \|\varphi(s)\|^2 ds\right)^2 + \left(\int\limits_{-\tau_M}^{0} d\theta \int\limits_{\theta}^{0} \|\dot{\varphi}(s)\|^2 ds\right)^2 + \\ &+ \left(N_d \int\limits_{-\frac{\tau_M}{N_d}}^{0} \|\varphi(s)\|^2 ds\right)^2 + \left(\int\limits_{-\frac{\tau_M}{N_d}}^{0} d\theta \int\limits_{\theta}^{0} \|\dot{\varphi}(s)\|^2 ds\right)^2 \end{aligned}}
$$

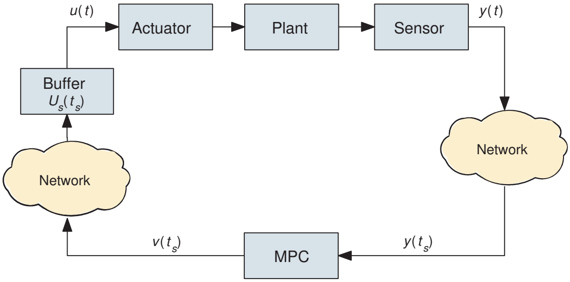


Figure 4.1: Configuration of considered NCS

## 4.2 Design of robust controller for NCS with packet-loss

The framework of NCS considered in the chapter is depicted in Fig. 4.1. Let the polytopic model of the plant to be controlled be described by the linear discrete time difference equation (3.1). Networks exist between sensor and controller and between controller and actuator. It is assumed that in network transmission there is negligible network-induced time delay (time delay is within sampling time of NCS) or it is treated as a dropout, but packet loss may happen. The sensor and the controller only send data at each sampling time, as well as the controller and actuator receive data. If data are lost at one sampling time, at next sampling time network only transmit new data and old data are discarded. The data are transmitted in a single packet. Base on (Xiong and Lam [2007]), the packet-loss process is redefined as follows. Let $\Im = \{t_1, t_2, ..., t_s, t_{s+1}, ...\}$ a subsequence of $\{1, 2, 3, ...\}$, denote the sequence of time points of successful data transmissions from the sensor to the actuator, and $l_{p\_max} = max(t_{s+1} - t_s - 1)$; $l_{p\_max} \leq N_u - 1$ be the maximum value of packet-loss number. Note that at time instant $t \in < t_s, t_{s+1} >$, if data is not successfully transmitted from the sensor to the controller, the controller will not calculate new control signal for the actuator and as result, the packet-loss occurs.

To overcome the resulting packet-loss problems, we use prediction based compensation schemes from (Grüne et al. [2009a];Grüne et al. [2009b]). Instead of a single input, a sequence of predicted future controls $U_s(t_s) = \{u(t_s), u(t_s + 1), ..., u(t_s + l_p), ..., u(t_s + l_{p\_max})\}$ is submitted and implemented at a buffer device with length $l_{p\_max}$ in the actuator. The buffer device is used to store the newest control sequence $U_s(t_s)$ transmitted successfully from MPC to actuator at sampling time $t_s \in \Im$. At time instant $t \in < t_s, t_s + l_p(t_s) >$, the packet loss occurs, and control action $u(t)$ corre-

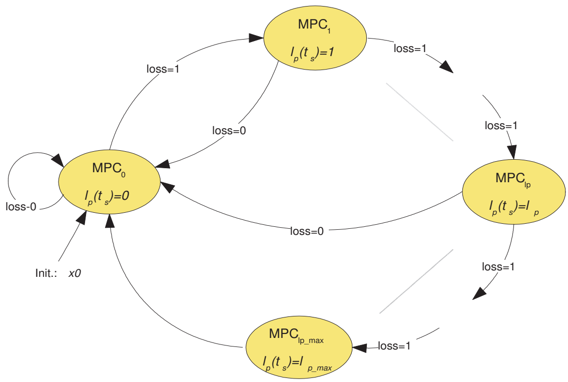


Figure 4.2: Schematic representation of a hybrid automaton

sponding to the current sampling time from control sequence $U_s(t_s)$ in the buffer device will be applied to the actuator. This process is considered as a switched system with schematic representation of a hybrid automaton in Fig.4.2.

Choose an appropriate output feedback predictive control algorithm as the controller of NCS as follows

$$u(t+k) = u(t+k|t) = \sum_{j=k}^{N_u} F_{kj}\left[y(t+j|t) - w(t+j|t)\right] \tag{4.9}$$

where the prediction is carried out over control horizon $N_u$ and prediction horizon $N_y = N_u$. The input control is constrained by (3.5). The main goal is to design a predictive controller (4.9) with input constraints so that, control action $u(t)$ from control sequence $U_s(t_s)$ robustly stabilizes NCS and ensures input constraints and guaranteed cost of the cost function (over the infinite optimization horizon).

Mechanism of the hybrid automaton can be represented as the following. At sampling time $t := t_s(l_p = 0)$, model predictive control $MPC_{l_p}$ defined as (3.7) is used to compute control sequence $U_s(t)$. If no packet loss at $t+1$, $MPC_0$ is applied. Otherwise, jump to $MPC_1$, it means that one packet is lost. Generally, at sampling time $t := t_s + l_p(1 \leq l_p \leq l_{p\_max})$, $MPC_{l_p}$ is applied. If no packet loss at $t+1$, jump to $MPC_0$. Otherwise, if $l_p < l_{p\_max}$, jump to $MPC_{l_p+1}$ and it means that $l_p + 1$ packets are lost.

The predicted states of $MPC_{l_p}$ for the instant $t+k, k = \{0, ..., N_u - 1\}$, are given by

$$\begin{array}{c} x(t+k+1|t) = A(\xi)x(t+k|t) + B(\xi)u(k) \\ u(k) = \begin{cases} u(t_s+l_p+k) & if \;\; 0 \le k \le N_u - l_p - 1 \\ u(t_s+N_u-1) & if \;\; N_u - l_p \le k \le N_u - 1 \end{cases} \end{array} \tag{4.10}$$

The closed-loop model prediction of $MPC_{l_p}$ is obtained as follows

$$A_{cf}(\xi, l_p)x_f(t+1) = A_{cx}(\xi, l_p)x(t) \tag{4.11}$$

where $A_{cf}(\xi, l_p) = A_f(\xi) - B_f(\xi)F_{l_pf}C_f$, $A_{cx}(\xi, l_p) = A_x(\xi) + B_f(\xi)F_{l_px}C$.

In the following theorem the novel formulation of robust stability condition is developed, which provide LMI for MPC robust stability analysis and BMI for MPC robust design.

**Theorem 4.3** *Control sequence $U_s(t_s)$ robustly stabilizes the NCS with loss packet process $\ell$ and ensures the guaranteed cost $J_0$, input constraints if and only if there exist matrices $H_{l_p} \in R^{nN_y \times n(N_y+1)}, P_{l_p}(\xi) = P_{l_p}^T(\xi) > 0$, gain matrices $F_{l_p}$, and scalars $\lambda_{i_d}^{l_p} \ge 0$ such that the following bilinear matrix inequality (BMI)*

$$W_{l_p}^{i_p}(\xi) = D_{l_p}^{i_p}(\xi) + A_m^T(\xi, l_p)H_{l_p}^{i_p} + {H_{l_p}^{i_p}}^T A_m(\xi, l_p) + Q + C_m^T F_{l_p}^T R F_{l_p} C_m \le 0 \tag{4.12}$$

*and the following linear matrix inequalities (LMIs)*

$$\begin{bmatrix} \widehat{P}_{l_p}(\xi) & * \\ D_{i_d}F_{l_p}C_m & \lambda_{i_d}^{l_p} \end{bmatrix} \ge 0 \;;\; \lambda_{i_d}^{l_p} \in \left\langle 0, \frac{\overline{u}_i^2}{\theta} \right\rangle \tag{4.13}$$

*hold for all $i_d = i + (j-1)m$, $i = \{1, ..., m\}$, $j = \{1, ..., N_u - l_p\}, i_p = \{0, l_p+1\}$, and $0 \le l_p \le l_{p\_max}$; where $D_{l_p}^{i_p}(\xi) = diag\{-P_{l_p}, P_{i_p} - P_{l_p}, ..., P_{i_p} - P_{l_p}, P_{i_p}\}(\xi) \in R^{n(N_y+1)\times n(N_y+1)}, A_m(\xi, l_p) = [A_{cx}(\xi, l_p) \quad -A_{cf}(\xi, l_p)]$, $\widehat{P}_{l_p}(\xi) = diag\{P_{l_p}(\xi), ..., P_{l_p}(\xi)\} \in R^{n(N_y+1)\times n(N_y+1)}$.*

Note that (4.12) is affine to $\xi$. If $W_{l_pj}^{i_p} \le 0, j = \{1, .., N\}$ is feasible with respect to unknown $P_{l_pj} = P_{l_pj}^T > 0, P_{i_pj} = P_{i_pj}^T > 0, H_{l_p}^{i_p}$, and $F_{l_p}$ for all $0 \le l_p \le l_{p\_max}, i_p = \{0, l_p+1\}$, then the control sequence $U_s(t_s)$ guarantees robust stability and guaranteed cost for NCS with predictive control (4.9) within the convex set. Therefore, BMI robust stability condition "if and only if" in (4.12) reduces to sufficient condition.

# 5 Conclusions

## 5.1 Contribution summary

The initial goal conducting this work was to investigate the robust control theories to design robust output feedback controller for uncertain system in the fields of decentralized control, model predictive control, and networked control system. Parameter Dependent Quadratic Stability criterion is used to synthesise robust controller with quadratic guaranteed cost for the polytopic description of the uncertain system. Based on the formulation of the dissertation's main objectives in Chapter 1, we can state the follows:

**Objective 1 -** Design of robust decentralized controller for large-scale system by using subsystem approach in state space was realized in Chapter 2. A new approach to design a robust decentralized output feedback PI controller for complex large-scale systems with a state decentralized structure was developed. The proposed design method is based on the Generalized Gershgorin Theorem and the LMI method to design robust PI controller guaranteeing feasible performance achieved in subsystems for the full system and therefore the proposed method excludes limit of system order in BMI solution. A robust decentralized PI controller has been designed using the polytopic description of the uncertain system and applying the robust optimal control design procedure including cost function to state-space subsystems generated in each vertex of the polytopic uncertainty domain. The main advantage of the proposed approach is that the order of the PI design procedure reduces to the order of the particular subsystem. Although the design procedure does not guarantee the stability of complex system, but it gives the way how the robust stability of complex system can be obtained. The effectiveness of the proposed method was illustrated by two examples such as three boiler-turbine subsystems and four cooperating DC motors. Based on numerical calculation and simulation results of these examples, the proposed control design scheme is believed to indicate the alternative in decentralized control, which can bring useful results.

**Objective 2 -** Design of robust output feedback model predictive control with input constraints to explicitly incorporate plant model uncertainty was realized in Chapter 3. An explicit MPC which contains all the possible combinations of uncertainty in the original-plant polytopic model, was employed. This MPC provides the robust stability guaranteeing for the uncertain plant model and uncertain model prediction. Additionally, an integrator is added to the controller design procedure to reject disturbances and maintain the process at the optimal operating conditions or setpoints. Two input constraints approaches such as heuristic one and invariant set are concerned.

The main contribution is that all the timedemanding computations of the output feedback gain matrices are realized off-line. The actual control value is obtained through simple on-line computation of scalar parameters and already computed feedback gain matrices. The numerical calculations and simulation results of two examples such as the double integrator and laboratory 3D-Crane plant were presented and they showed that, the effectiveness of the proposed method, namely its ability to cope with robust stability, successfully to force disturbance rejection and setpoint tracking and input constraints without complex computational load, were obtained.

**Objective 3 -** Design of robust controller for uncertain NCS with time-varying network-induced delay was realized in Section 4.1 of Chapter 4. Two new approaches such as complete LKF and discretized LKF to design robust output feedback PID controllers achieving a guaranteed cost such that the NCSs can be stabilized for all admissible polytopic-type uncertainties and time-varying delays with less conservatism than previous works were presented. In the discretized LKF approach, a partitioning scheme of time-varying delay and IQC are used to overcome the conservatism in the output feedback design procedures. The effectiveness as well as conservatism of algorithm was showed by comparing results obtained by the discretized LKF method, where controller has been designed for partitioning of time-delay $N_d = \{1, 2, 3, 5\}$ with the complete LKF method on 1000 randomly generated examples. The numerical calculation results show that increasing the number of parts $N_d$, to which the time-delay interval is divided, reduces the conservatism of robust stability condition, therefore the number of successful controller designs increases with the increased $N_d$.

**Objective 4 -** Design of robust predictive controller for uncertain NCS with arbitrary packet-loss was realized in Section 4.2 of Chapter 4. A robust output feedback linear model predictive control scheme over a network with double-sided packet loss was implemented. This one is built based on the combination of compensation mechanism and robust model predictive control design approach in Chapter 3. As a result, networked predictive control systems with loss packet are modeled as switched linear systems and the robust stability condition of networked model predictive control with packet dropout is established by using the theory of switched systems. The proposed method was evaluated by a numerical example-uncertain double integrator controlled over network. Networked MPC with 8-ahead steps prediction was investigated and it means that up to 87.5% of the packets can be lost during the network transmissions. Simulation was realized in time interval $100[s]$, and in this simulation interval time, packet-loss process was generated randomly with 66.318% of packets lost. The simulation results showed that, the NCS was robustly stable and guaranteed input constrained for model uncertainty.

Finally, we would like to conclude that, all predefined goals of dissertation are obtained and the results presented in dissertation were published in many journals and conferences. For more details, see Publications by author.

## 5.2 Future work

Besides the encouraging results, there are still some open questions as well as interesting ideas raised during the course of our research but have not exploited further in the scope of this dissertation for some reasons.

### Enhancing feasibility of optimization problem solution

One of the most important issues raised is the feasibility or convergence of optimization problem solution due to its size. Since the controller designs require numerical solution of BMI, and the size of the optimization problem significantly increases with increased time-delay partition $N_d$ or number of packet loss $l_p$ or number of step prediction $N_y$ $(N_u)$, this limits the choice of increasing $N_d; l_p; N_y$ $(N_u)$. As the result, it will restrict the effectiveness of proposed design procedures especially for large-scale systems. Based on our view, solution of this issue can be realized by reforming the optimization problem in the form of LMI instead of BMI; and in the case of large-scale system, a combining this LMI optimization problem with appropriate decentralized strategy (subsystem approach) is required.

### Robust distributed MPC

Our robust MPC was typically implemented in a centralized way. The complete system was modeled, and all the control inputs were computed in one optimization problem. In large-scale applications, such as power systems, water distribution systems, traffic systems, manufacturing systems, networked systems and economic systems, it is useful to have distributed or decentralized control schemes, where local control inputs are computed using local measurements and reduced-order models of the local dynamics. Therefore, it would be interesting to implement the robust MPC in a decentralized (distributed) fashion.

### Multiple-packet transmission in NCS

In the dissertation, we assumed that, the capacity of communication network is unlimited. Therefore, the control sequence is encapsulated into a single packet and transmitted over network. However, the practice is the bandwidth and packet size constraints of the network and thus the data may be split into separate packets before transmitting, so-called multiple-packet transmission. This property makes synthesis of robust controller for NCS with packet-loss by using prediction based compensation schemes more challenging.

# Bibliography


Y. Ariba and F. Gouaisbaut. Delay-dependent stability analysis of linear systems with time-varying delay. *IEEE Conference on Decision and Control*, pages 2053–2058, 2007. 14

B. Azimi-Sadjadi. Stability of networked control systems in the presence of packet losses. *In: Proceedings of the conference on decision and control*, Hawaii, USA: 676–681, 2003. 5

W.H. Chen, Z.H. Guan, and X. Lu. Delay-dependent output feedback guaranteed cost for uncertain time-delay systems. *Automatica*, 40:1263–1268, 2004. 4

B. Ding. Stabilization of linear systems over networks with bounded packet loss and its use in synthesizing model predictive control. *In: Proceedings of 8th, International Conference on Control and Automation*, Xiamen, China:2258–2263, 2010b. 5

E. Friedman and S.J. Niculescu. On complete lyapunov-krasovskii functional techniques for uncertain systems with fast varying delays. *International Journal of Robust and nonlinear Control*, 18:364–374, 2008. 4

L. Grman, D. Rosinová, V. Veselý, and A. Kozáková. Robust stability conditions for polytopic systems. *Int. Journal of Systems Science*, 36(15):961–973, 2005. 2

L. Grüne, J. Pannek, and K. Worthmann. A networked unconstrained nonlinear mpc scheme. *In: Proceedings of the European Control Conference*, pages 91–96, 2009a. 5, 16

L. Grüne, J. Pannek, and K. Worthmann. A prediction based control scheme for networked systems with delays and packet dropouts. *In: Proceedings of the 48th IEEE CDC*, Shanghai, China:537–542, 2009b. 5, 16

K. Gu, V.L. Kharitonov, and J. Chen. Stability of time-delay systems. *Control engineering, Birkhauser Boston*, 2003. 4

M.V. Kothare, V. Balakrishnan, and M. Morari. Robust constrained model predictive control using linear matrix inequalities. *Automatica*, 32:1361–1379, 1996. 3

A. Kozaková, V. Veselý, and J. Osuský. In: Decentralized controllers design for performance: Equivalent subsystems method. *European Control Conference, ECC09. Budapest, Hungary, 23-26 August*, pages CD–ROM, 2009. 3

B. Lee and J. Gyu Lee. Robust stability and stabilization of linear delayed systems with structured uncertainty. *Automatica*, 35:1149–1154, 1999. 4

Z. J. Li, D. H. Sun, Y. T. Shi, and L. F. Wang. A stabilizing model predictive control for network control system with data packet dropout. *Journal of Control Theory and Application*, 7(3):281–284, 2009. 5

M.C. De Olivera, J. Bernussou, and J.G. Geromel. A new discrete-time robust stability condition. *Systems and Control Letters*, 37:261–265, 1999. 2

D. Peaucelle, D. Arzelier, O. Bachelier, and J. Bernussou. A new robust d-stability condition for real convex polytopic uncertainty. *Systems and Control Letters*, 40: 21–30, 2000. 2

Y. M. Repin. Quadratic lyapunov functional for systems with delay. *Prikl. Mat. Meh(Russian)*, 29:564–566, 1965. 4

B. Rohal-Ilkiv. A note on calculation of polytopic invariant and feasible sets for linear continuous -time systems. *Annual Review in Control*, 28:59–64, 2004. 12

D. Rosinová and V. Veselý. Robust pid decentralized controller design using lmi. *1sth IFAC Workshop on Applications of Large Scale Industrial Systems. Helsinki,Finland, 30-31Aug*, 2006. 3

D. Rosinová, V. Veselý, and V. Kučera. A necessary and sufficient condition for static output feedback stabilizability of linear discrete-time systems. *Kybernetika*, 39(4):447–459, 2003. 2

S.S. Stankovič, D.M. Stipanovič, and D.D. Šiljak. Decentralized dynamic output feedback for robust stabilization of a class of nonlinear interconnected systems. *Automatica*, 43:861–867, 2007. 3

V. Veselý and R. Bars. Stable output feedback model predictive control design: Lmi approach. *Archives of Control Sciences*, 18:385–394, 2008. 4

V. Veselý and D. Rosinová. Robust output model predictive control design: Bmi approach. *ICIC International*, 5:15–23, 2009. 4, 6

V. Veselý, D. Rosinová, and M. Foltin. Robust model predictive control design with input constraints. *ISA Transactions*, 49:114–120, 2010. 4, 6, 12

Y. Xia, J. Chen, P. Shi, G. Liu, and D. Rees. Guaranteed cost and positive real control of uncertain systems via static output feedback. *International Journal of Innovative Computing, Information and Control*, 4(6):1275–1282, 2008. 4

J. Xiong and J. Lam. Stabilization of linear systems over networks with bounded packet loss. *Automatica*, 43:80–87, 2007. 5, 16

L. Yu and J. Chu. An lmi approach to guaranteed cost control of linear uncertainty time-delay systems. *Automatica*, 35:1155–1159, 1999. 4

A.I. Zečevič and D.D. Šiljak. Design of robust static output feedback for large-scale systems. *IEEE Trans. on Automatic Control*, 49(11):2040–2044, 2004. 3

W. Zhang, M. S. Branicky, and S. M. Philip. Stability of networked control systems. *IEEE Control System Magazine*, 21(1):84–89, 2001. 5

# Publications by author

## Journals


[1] Q.T. Nguyen, I. Holič and V. Veselý. Robust Guaranteed Cost Controller Design for Networked Control Systems: Discretized Approach. *International Journal of Innovative Computing, Information and Control*, 7(6): 3533-3543, 2011. (ISI indexed, IF = 1.667)

[2] Q. T. Nguyen, V. Veselý, and D. Rosinová. Design of robust model predictive control with input constraints. *International Journal of Systems Science*, 2011, Accepted. (ISI indexed, IF = 0.948)

[3] V. Veselý and Q.T. Nguyen. Robust Output Networked Control System Design. *ICIC Express Letters*, 4(4): 1399-1404, 2010.

[4] Q.T. Nguyen and V. Veselý. Design of Robust Guaranteed Cost PID Controller for Networked Control Systems. *Journal of Electrical Engineering*, 61(2): 114-119, 2010.

[5] Q.T. Nguyen and I. Holič. Robust Controller Design for the 3DCrane Process. *AT&P Journal Plus*, number 1:Systémy automatického riadenia: 95-99, 2010.

[6] Q.T. Nguyen and V. Veselý: Robust Guaranteed Cost PID Controller Design for Networked Control Systems (NCSs). *Journal of Cybernetics and Informatics*, 10: 1-10, 2010.

[7] V. Veselý and Q.T. Nguyen. Robust Decentralized Controller Design for Large Scale Systems: Subsystems Approach. *Acta Montanistica Slovaca*, 16(1): 84-88, 2011.

[8] V. Veselý and Q.T. Nguyen. Robust Power System Stabilizer via Networked Control System. *Journal of Electrical Engineering*, 62(5): 386-291, 2011.

[9] D. Rosinová, Q.T. Nguyen, V. Veselý and L'ubomír Marko. Robust Decentralized Controller Design: Subsystem Approach. *Journal of Electrical Engineering*, 63(1), 28-34, 2012.


## International Conferences and Workshops


[10] Q.T. Nguyen and V. Veselý. Design of Robust Networked Predictive Control Systems with Packed Loss. In: *4th IFAC Nonlinear Model Predictive Control Conference*, Noordwijkerhout, Netherland, 2012. Accepted.

[11] V. Veselý, D. Rosinová and Q.T. Nguyen. Robust Networked Output Feedback Predictive Controller Design. In: *7th IFAC Symposium on Robust Control Design*, Aalborg, Denmark, 2012. Accepted.

[12] Q.T. Nguyen. Robust Time Delay Controller Design Using Discretized Lyapunov Krasovskii Functionals. In: *Selected Topics on Constrained and Nonlinear Control*, 257-263, 2011.

[13] Q.T. Nguyen, I. Holič and V. Veselý. Robust Guaranteed Cost Controller Design for Networked Control Systems: Discretized Approach. In: *Process Control 2010 : 9th International Conference*, Kouty nad Desnou, Czech Republic, C114b [12], 2010.

[14] Q.T. Nguyen, V. Veselý and D. Rosinová: Robust Model Predictive Control Design. In: *PEIT '011. New Paradigms in Electronics and Information Technologies*, Alexandria, Egypt,CD-Rom, 2011.

[15] V. Veselý and Q.T. Nguyen: Robust Decentralized Controller Design for Large Scale Systems. In: *ICCC 2011 : 12th International Carpathian Control Conference*, Velké Kralovice, Czech Republic,429-432, 2011.

## Local and National Conferences

[16] I. Holič and Q.T. Nguyen. Robust Stabilization of Polytopic Systems: Pole Placement Based on LMI Regions with Input Constraints. In: *Kybernetika a informatika 2012*,Skalka pri Kremnici, slovak Republic, 27-28, 2012.

[17] A. Kozaková, V., Veselý and Q.T. Nguyen. Robust Decentralized Controller Design: State Space Version of the Equivalent Subsystems Method. In: *Kybernetika a informatika 2010*, Vyšná Boca, Slovak Republic, CD-Rom, 2010.

[18] Q.T. Nguyen and I., Holič: Design of Robust PI Controller with Input Constraints. In: *Kybernetika a informatika 2012*, Skalka pri Kremnici, Slovak Republic,95-96, 2012.

[19] Q.T. Nguyen, A. Kozaková and V., Veselý. Robust Decentralized Controller Design Using Equivalent Subsystems Approach. In: *Power Engineering 2010*, Tatranské Matliare, Slovak Republic, CD-Rom[7], 2010.

[20] Q.T. Nguyen and V., Veselý. Robust Decentralized PID Controller Design for the 3D Crane Process. In: *Process Control 2011 : Proceedings of the 18th International Conference*. Tatranská Lomnica, Slovakia, 485-489, 2011.

[21] Q.T. Nguyen and V., Veselý. Robust Guaranteed Cost PID Controller Design for Networked Control Systems (NCSs).In: *Kybernetika a informatika 2010*, Vysná Boca, Slovak Republic, CD-Rom, 2010.

[22] Q.T. Nguyen. Robust Model Predictive Control Design. In: *ELITECH11 : 13th Conference of Doctoral Students Faculty of Electrical Engineering and Information Technology*, Bratislava, Slovak Republic, 1-8, 2011.

[23] V. Veselý and Q.T. Nguyen. Power System Stabilizer via Networked Control System. In: *Power Engineering 2010*, Tatranské Matliare, Slovak Republic, CD-Rom[10], 2010.