

Ing. Róbert Krasňanský

Summary of doctoral dissertation

HYBRID CONTROL STRUCTURES OF COMPLEX MECHATRONIC SYSTEMS

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Author: Ing. Róbert Krasňanský
Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology, Ilkovičova 3, 812 19 Bratislava

Supervisor: assoc. prof. Ing. Alena Kozáková, PhD.
Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology, Ilkovičova 3, 812 19 Bratislava

Opponents: prof. Ing. Boris Rohal'-Ilkiv, CSc.
Institute of Automation, Measurement and Applied Informatics, Faculty of Mechanical Engineering, Nám. slobody 17, 812 31 Bratislava 1

assoc. prof. Peter Ševčík, PhD.
Department of Technical Cybernetics, Faculty of Management Science and Informatics, University of Žilina, Univerzitna 1, 010 26 Žilina

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prof. Dr. Ing. Miloš Oravec
Dean of FEI STU

Abstract

Thesis Title: Hybrid control structures of complex mechatronic systems

Keywords: Switched systems, Frequency domain, Stability condition, Robust control, Decentralized control, Hardware implementation

This thesis deals with development of hybrid control structures for complex mechatronic systems in the frequency domain and their algorithmization for the purposes of hardware implementation on FPGAs. The objective of the thesis is to develop the robust control design procedure guaranteeing closed-loop stability and nominal performance for SISO and MIMO switched systems in the frequency domain. New switched system stability condition is developed based on the small gain theorem to guarantee closed-loop stability of the multi-model plant in all operation modes as well as stability during arbitrary switching between individual operation modes. For multivariable systems a design approach based on the previous is derived. The applied decentralized control structure is based on the Equivalent Subsystems Method, which allows to design local SISO controllers guaranteeing nominal performance of the full system. For uncertain switched systems a robust control design procedure guaranteeing robust stability in individual operation modes. Local controllers can be calculated using any frequency design method. This thesis also deals with a development of control design procedure for complex systems with multiple inputs and outputs based on identification of equivalent subsystems and independent design of local SISO predictive controllers while considering given performance specifications for individual subsystems. Presented control approaches serve as a basis for design of control algorithms and their hardware implementation on FPGA platforms. The respective control algorithms are developed using VHDL language and experimentally tested on real laboratory plants.

Anotácia dizertačnej práce

Názov dizertačnej práce: Hybridné štruktúry riadenia zložitých mechatronických systémov

Kľúčové slová: Systémy s prepínaním, Frekvenčná oblasť, Podmienka stability, Robustné riadenie, Decentralizované riadenie, Hardvérová implementácia

Cieľom práce je vývoj postupov pre návrh robustného riadenia garantujúceho stabilitu uzavretého regulačného obvodu a kvalitu pre SISO a MIMO systémy s prepínaním vo frekvenčnej oblasti. Použitím teórie malého zosilnenia je odvodená nová podmienka stability pre systémy s prepínaním, ktorá garantuje stabilitu uzavretého regulačného obvodu vo všetkých režimoch ako i počas prepínania medzi jednotlivými režimami. Pre mnohorozmerné systémy je odvodená metóda návrhu založená na predchádzajúcich prácach. Použitá decentralizovaná štruktúra riadenia je založená na metóde ekvivalentných podsystémov umožňujúcej návrh lokálnych SISO regulátorov s predpísanou kvalitou riadenia ktorá je garantovaná i pre plný systém. Pre systémy s neurčitostami je odvodený postup návrhu robustného riadenia pre systémy s prepínaním zabezpečujúci robustnú stabilitu v jednotlivých režimoch systému. Na návrh lokálnych regulátorov je možné použiť ľubovoľnú frekvenčnú metódu. Práca sa ďalej zaoberá vývojom metodiky návrhu decentralizovaného riadenia pre zložité mnohorozmerné systémy založenom na identifikácii ekvivalentných podsystémov a nezávislým návrhom lokálnych SISO prediktívnych regulátorov pri uvažovaní daných požiadaviek na kvalitu riadenia v rámci jednotlivých regulátorov. Prezentované metódy riadenia slúžia ako základ pre návrh algoritmov a ich hardvérovú implementáciu na FPGA štruktúrach. Príslušné algoritmy riadenia sú napísané v jazyku VHDL a testované experimentálne na reálnych laboratórnych procesoch.

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1 Introduction

One of the most important concepts in control design theory is the model of the system. Basically, a model is an abstract, simplified representation of the real system with the sufficient degree of complexity to describe the system behavior. Traditionally, a model of the system is derived from known physical laws or based on the time evolution of system parameters estimated from experimental measurements. The study of dynamical systems has been traditionally focused on two domains, systems continuous and discrete in time.

Recent years have witnessed an enormous growth of interest in dynamic systems that are characterized by a mixture of both continuous-time and discrete-time dynamics. Such systems are commonly found in engineering practice, referred to as hybrid or *switched systems*. The widespread application of such systems is motivated by ever increasing performance requirements and reducing complexity which was and still is an important reason for dealing with hybrid systems. This can be accomplished by switching between relatively simple linear time-invariant (LTI) systems. In control engineering, switching between simpler dynamical systems has been successfully used in practice for many decades. However, the potential gain of switched systems is offset by the fact that the switching action introduces a specific behavior in the overall system which is not present in any of the composite subsystems. For example, it can be easily shown that switching between stable subsystems may lead to instability or chaotic behavior of the overall system. On the other hand, switching between unstable subsystems may result in a stable overall system. Recent efforts in hybrid systems research along these lines typically concentrate on the analysis of the dynamic behaviors and design of controllers with guaranteed stability and performance.

Switching between a number of control structures automatically results in control systems that are no longer constrained by limitations of linear design. It is therefore not surprising that switching-based control strategies might result in algorithms offering significant performance improvements over traditional linear control. For example, different controllers may be encoded within a single structure resulting in a control system with enhanced functionality by exploiting the benefits of each of the constituent controllers.

Most of the results presented in the literature is dealing with the time-domain approaches, however there are very little results developed in the frequency domain. Although most of the time-domain methods provide successful results, the computation effort may become very high in the case of higher order systems and therefore makes the solution to be very difficult. The frequency-domain approaches are on the other hand very attractive to engineering community due to the easy and more visual computation as well as implementation of the controllers. Thus, the research in this domain may bring new ideas and methods, which might become a worthy competitors to other time domain approaches.

Complex engineering systems are often designed in a decentralized manner. Each component subsystem is usually designed in relative isolation, and the overall system is constructed by combining the subsystems by means of some

appropriate supervisory logic. Multivariable or multi-loop control takes into consideration interactions between loops which improves control performance unlike the use of multiple single loop controllers, which represents the simplest, however often not satisfactory solution. In many cases this approach leads to switched linear control systems. The main advantage of a decentralized control scheme is then in splitting of the whole control problem into several local control blocks which only acquire local output measurements and calculation of local control inputs, possibly supervised by some upper hierarchical control layer. This leads to parallel computations and reduced communications, although all controllers involved in controlling the process must be designed in regard to conditions under which they stabilize the entire system. Besides advantages in controller implementation (parallel computation, reduced communications) a big advantage of decentralized control is maintenance: while certain parts of the overall process are interrupted, the remaining parts keep operating (possibly with reduced performance) in closed-loop with their local controllers without the need of stopping the overall process as in case of centralized control. Moreover, a partial redesign of the process does not necessarily implies a complete redesign of the controller as it would in case of centralized control.

Uncertainties in modeling of the complex dynamic behavior of the complex systems together with high dimensions make a control problem much more difficult even using the centralized control structure. Therefore the need for design of robust decentralized control schemes for applications in multivariable systems is actual, as well as investigating the robust stability of interconnections of the subsystems with local controllers.

The widespread availability of low cost digital platforms such as microcontrollers have allowed digital system implementation to evolve into more flexible digital form, exclusively employed in present applications (Paraskevopoulos, 1996; Forsythe and Goodall, 1991). Motivated by the practical success of conventional control methods applied in industrial process control, there has been an increasing amount of work on development of effective hardware realizations of control algorithms.

Recently, it has been shown that Field Programmable Gate Arrays (FPGAs) can pose an alternative solution for the realization of digital control systems, offering re-programmable hardware logic with greater flexibility from the logic complexity and development point of view. It allows designers to develop a fully hardware architecture which is dedicated to the control algorithm to be implemented. This leads to specific advantages to the controller, since the FPGAs offers true parallel processing capabilities, which eliminate the issue of the computational delay associated with the sequentially executing microprocessors or Digital Signal Processors (DSPs). Moreover, increasing density of FPGAs along with their high degree of flexibility pushes designers to employ them for designing controllers used in a large range of industrial applications. Thus, FPGAs provide a promising architectures for controllers implementation, making this area very attractive and perspective for control engineers.

1.1 Objectives of the Thesis

Based on the review of existing design approaches and identified open issues we can conclude that compared to time-domain design methods, frequency-domain control design methods are better understood in control engineering community, since it provide comprehensible insight on important concepts (e.g. bandwidth and closed-loop peaks).

The primary aim of this work is to develop new application-oriented hybrid control structures and approaches to the robust and decentralized control design in the frequency domain for complex systems involving hybrid (switched) systems and dynamical systems with multiple input and multiple outputs which will be suitable for hardware implementation on FPGA platforms to control processes with fast dynamics.

The main objectives of the thesis can be summarized as follows:

1. Switched system controller design in the frequency domain.
 - Development of control design procedure for switched systems in the frequency domain guaranteeing closed-loop stability in individual operation modes as well as stable switching between operation modes, which will be applicable for both SISO and MIMO systems.
 - Extension of the design procedure for switched systems to guarantee robust stability in individual operation modes.
2. Decentralized model predictive control design.
 - Development of decentralized model predictive control design procedure for complex systems with multiple inputs and outputs based on Equivalent Subsystem Method.
3. Verification of the proposed theoretical results on a series of examples and case studies.
4. Hardware implementation of developed control algorithms.
 - Algorithmization of the proposed control design methods for hardware realization using FPGAs.
 - Hardware implementation of developed control algorithms on FPGA platform.
 - Experimental verification and application of implemented control algorithms on real laboratory plants.

2 Frequency-Domain Robust Switched System Controller Design

A novel frequency-domain stability condition for continuous-time SISO and MIMO switched systems represented in the affine form is presented. It is based on the $M - \Delta$ structure and small gain theorem. The switched controller design procedure is derived for closed-loop stability in all operation modes as well as for stable switching between them. By employing Equivalent Subsystems Method (ESM) (Kozáková, 2012), the design procedure is enhanced to guarantee robust stability of the closed-loop system in individual operation modes allowing design of robust decentralized controllers for SISO and MIMO plants.

Consider a MIMO (SISO) multi-model plant $\bar{G}(s)$ with m inputs and m outputs ($m \geq 1$) which switches between p operation modes in the affine form (Kozáková, Veselý, and Krasňanský, 2014)

$$\bar{G}(s) = G_{00}(s) + \sum_{i=1}^p G_i(s)q_i, \quad q_i \in I_q \quad (2.1)$$

where $\bar{G}(s)$, $G_{00}(s)$, $G_i(s)$ are known $m \times m$ transfer function matrices, q_i is a scalar, which indicates the arbitrary switching signal and I_q denotes the finite set of indices given as follows

$$I_q = \left\{ q \in p : q_i = \begin{cases} 1 \\ 0 \end{cases}, \quad i = 1, 2, \dots, p \right\} \quad (2.2)$$

where $q_i = 1$ indicates when the switched system is described by the i -th operation mode, otherwise $q_i = 0$. It is assumed that $\sum_{i=1}^p q_i = 1$. The switched controller $\bar{R}(s) \in \mathbb{R}^{m \times m}$ is designed in a similar affine form

$$\bar{R}(s) = R_{00}(s) + \sum_{i=1}^p R_i(s)q_i, \quad q_i \in I_q \quad (2.3)$$

where $\bar{R}(s)$, $R_{00}(s)$, $R_i(s)$ are known $m \times m$ transfer function matrices. The multi-model system (2.1) and the corresponding switched controller (2.3) can be written in the matrix-vector form

$$\bar{G}(s) = G_{00}(s) + Q^T G(s) \quad (2.4)$$

$$\bar{R}(s) = R_{00}(s) + Q^T R(s) \quad (2.5)$$

where

$$G(s) = \begin{bmatrix} G_1(s) \\ \vdots \\ G_p(s) \end{bmatrix}, R(s) = \begin{bmatrix} R_1(s) \\ \vdots \\ R_p(s) \end{bmatrix}, Q = \begin{bmatrix} q_1 I_{m \times m} \\ \vdots \\ q_p I_{m \times m} \end{bmatrix} \quad (2.6)$$

$\bar{G}(s)$, $\bar{R}(s)$ are $pm \times m$ transfer function matrices, q_i , $i = 1, 2, \dots, p$ are scalars, $G(s)$, $R(s)$ are $pm \times m$ transfer function matrices and Q is $pm \times m$ matrix

consisting of stacked $m \times m$ identity matrices, which reflects switching between individual operation modes and corresponding controllers. Substituting transfer function matrices (2.4) - (2.5) into standard feedback loop the feedback configuration in Fig. 1 is obtained.

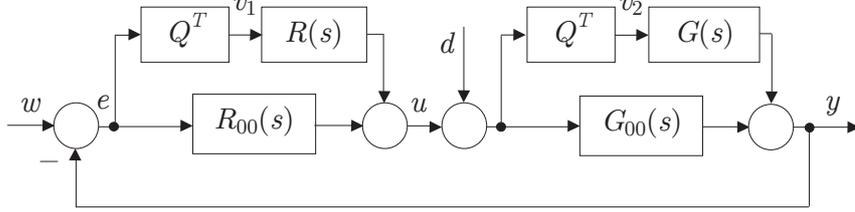


Figure 1: Switching feedback loop

Using the notation

$$\begin{aligned} y_g &= M_g(s)u_g \\ y_g &= \begin{bmatrix} e \\ u \end{bmatrix}, \quad u_g = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \end{aligned} \quad (2.7)$$

a new $M_g - Q_g$ structure (Fig. 2) is obtained based on the input-output model derived from Fig. 1, where

$$M_g(s) = \begin{bmatrix} -R(s) \frac{G_{00}(s)}{I + G_{00}(s)R_{00}(s)} & -G(s) \frac{1}{I + G_{00}(s)R_{00}(s)} \\ R(s) \frac{1}{I + R_{00}(s)G_{00}(s)} & -G(s) \frac{R_{00}(s)}{I + R_{00}(s)G_{00}(s)} \end{bmatrix} \quad (2.8)$$

$$M_g(s) \in \mathbb{RH}_{\infty}^{2m \times 2pm}$$

$$Q_g = \begin{bmatrix} Q^T & 0 \\ 0 & Q^T \end{bmatrix}, \quad Q_g \in \mathbb{R}^{2pm \times 2m} \quad (2.9)$$

Theorem 1. (Small Gain Theorem) *Suppose that the transfer function matrix $M_g(s)$ is a strictly bounded real transfer function matrix and $Q_g \in \Phi_{br}$ represents time-varying memoryless nonlinearity. Then the feedback interconnection in Fig. 2 is asymptotically stable.*

The proof can be found in [Haddad and Bernstein \(1991\)](#). Considering the spectral matrix norm

$$\sigma_{max}[Q_g] = \sqrt{(Q_g^T Q_g)} = \sqrt{\sum_{i=1}^p q_i^2} = 1 \quad (2.10)$$

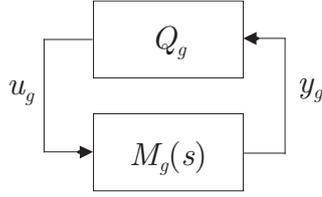


Figure 2: $M_g - Q_g$ structure

the closed-loop system stability condition

$$\sigma_{max}[M_g(j\omega)] < \frac{1}{\max_q \sigma_{max}(Q_g)}, \quad \forall \omega \quad (2.11)$$

is modified to strictly bounded realness of $M_g(s)$

$$\sigma_{max}[M_g(j\omega)] < 1, \quad \forall \omega \quad (2.12)$$

where parameters $q_i \in \{0, 1\}$ and $\sum_{i=1}^p q_i = 1$. Moreover, by factorization of the matrix (2.8) into the following form

$$M_g(s) = M_1(s)M_2(s) \quad (2.13)$$

where

$$M_1(s) = \begin{bmatrix} -\frac{G_{00}(s)}{I + G_{00}(s)R_{00}(s)} & -\frac{1}{I + G_{00}(s)R_{00}(s)} \\ \frac{1}{I + R_{00}(s)G_{00}(s)} & -\frac{R_{00}(s)}{I + R_{00}(s)G_{00}(s)} \end{bmatrix} \quad (2.14)$$

$$M_2(s) = \begin{bmatrix} R(s) & 0 \\ 0 & G(s) \end{bmatrix} \quad (2.15)$$

the stability condition for switched closed-loop system can be expressed in the form

$$\sigma_{max}[M_1(j\omega)] < \frac{1}{\sigma_{max}[M_2(j\omega)]} \quad \forall \omega \quad (2.16)$$

2.1 Switched Controller Design Procedure:

The frequency-domain switched controller design procedure for arbitrary switching results from the above development in the following order:

1. From mathematical models in individual operation modes $\bar{G}^i(s)$, $i = 1, \dots, p$ the affine model of the switched system (2.1) is generated.
 - $G_{00}(s)$ is calculated e.g. as a mean value parameter model

$$G_{00}(s) = \sum_{i=1}^p \bar{G}^i(s)/p \quad (2.17)$$

- $G(s)$ is calculated from differences between the nominal model and models in individual operation modes

$$G_i(s) = G_{00}(s) - \bar{G}^i(s), \quad i = 1, \dots, p \quad (2.18)$$

2. For each operation mode a controller $\bar{R}^i(s)$, $i = 1, \dots, p$ is designed using any frequency domain design method. It is recommended to use a design method, which allows output performance specification.
3. From controllers designed for individual operation modes, the controller in the affine form (2.3) is generated.

- $R_{00}(s)$ can be obtained in two ways:
 - calculated as a mean value parameter controller

$$R_{00}(s) = \sum_{i=1}^p \bar{R}^i(s)/p \quad (2.19)$$

- or designed for $G_{00}(s)$ such that it ensures stability of characteristic equation $\det(I + G_{00}(s)R_{00}(s))$.
- assuming $G_{00}(s)$ and $G(s)$ are stable, $R(s)$ is calculated from differences between $R_{00}(s)$ and the controllers designed for individual operation modes.

$$R_i(s) = R_{00}(s) - \bar{R}^i(s), \quad i = 1, \dots, p \quad (2.20)$$

4. Closed-loop stability of the switched system is verified if the condition (2.12) or (2.16) is met.

- if the stability condition is not passed, the algorithm start again from step 2 and controllers $\bar{R}^i(s)$, $i = 1, \dots, p$ are designed with tuning parameters ensuring better stability properties.

For MIMO systems the controller design is more complicated due to the interactions. This problem is dealt using the ESM approach. The second step in design procedure consists of the following:

- splitting of the full matrix $\bar{G}^i(s)_{m \times m}$ of the model (2.1) in each operation mode into diagonal $G_d^i(s)$ and off-diagonal part $G_w^i(s)$, $i = 1, \dots, p$.
- calculation of characteristic loci $g_v^i(j\omega)$, $v = 1, \dots, m$; $i = 1, \dots, p$ of the matrix of interactions $G_w^i(s)$
- generating m equivalent subsystems $G_v^{eq(i)}(s)$, $i = 1, \dots, p$; $j = 1, \dots, m$ for each operation mode using a selected fixed $g_k^i(j\omega)$ from the m characteristic loci $g_v^i(j\omega)$.
- Design of local robust SISO controllers independently for each equivalent subsystem using frequency-domain control design method ensuring required performance specifications.

The following steps remain the same.

2.1.1 Design Procedure for Robust Stability

Consider an uncertain system described by any unstructured uncertainty with $m \geq 1$ subsystems given by a set of M transfer function matrices obtained by identification in M working points for each mode i , where

$$G_l^i(s) \in \Pi, \quad i = 1, 2, \dots, p; \quad l = 1, 2, \dots, M \quad (2.21)$$

represents a member of a set of possible plants. The nominal model in each operation mode i is calculated as a mean value parameter model. Then the overall design procedure is enhanced by the several initial steps to guarantee robust stability in individual operation modes.

- Calculation of scalar weight function for selected unstructured uncertainty model (for additive uncertainty)

$$\ell^i(\omega) = \max_l \sigma_{max} \left\{ G_l^i(j\omega) - \bar{G}_0^i(j\omega) \right\}, \quad l = 1, 2, \dots, M; \quad i = 1, \dots, p \quad (2.22)$$

- Calculation of upper bound for the nominal equivalent complementary sensitivity function (for additive uncertainty)

$$\sigma_{max} \left(T_0^i(j\omega) \right) < \frac{\sigma_{min} [\bar{G}_0^i(j\omega)]}{\ell_a^i(\omega)} = L_a^i(\omega), \quad \forall \omega \quad (2.23)$$

or eventually for the nominal equivalent sensitivity function (for inverse additive uncertainty)

$$\sigma_{max} \left(S_0^i(j\omega) \right) < \frac{1}{\ell_{ia}^i(\omega) \sigma_{max} [\bar{G}_0^i(j\omega)]} = L_{ia}^i(\omega), \quad \forall \omega \quad (2.24)$$

Since the expressions on the r.h.s of (2.23) and (2.24) do not depend on any particular controller, they can be evaluated prior to controller design. The respective relations for other uncertainty models (input/output multiplicative) can be found in [Kozáková and Veselý \(2013\)](#).

- Performance specification in terms of minimal phase margin for equivalent subsystems

$$PM_i \geq 2 \arcsin \left(\frac{1}{2 \min_{\omega} (L_k^i(\omega))} \right) \geq \frac{1}{\min_{\omega} (L_k^i(\omega))} [\text{rad}] \quad (2.25)$$

where $i = 1, \dots, p$; $k \in \{a, i, ia\}$. The design procedure now continues from 1. step according to Section 2.1. The robust controller design can be performed as follows:

- for SISO systems any robust design approach can be used, e.g. the Edge Theorem and the Neymark D-partition method ([Hypiusová and Osuský, 2010](#)), Small Gain Theorem ([Bhattacharyya, Chapellat, and Keel, 1995](#)) or Bode design method ([Bucz et al., 2010](#)).
- for MIMO systems the ESM approach is used. Local robust SISO controllers are designed independently for each equivalent subsystem using design method ensuring required performance specifications.

The final step is enhanced by verification of the robust stability condition (2.23) or (2.24) in individual operation modes.

2.2 Example

The practical application of the proposed control design technique was tested on a MIMO plant with two inputs and two outputs. Two operation modes of the plant are considered. In each operating mode, mathematical model of the plant $\bar{G}_l^i(s)$ has been experimentally identified from I/O data measured in 3 working points ($l = 1, \dots, M$) given by different values of loads.

Obtained mean value parameter nominal models in each operation mode $\bar{G}^i(s)$, $i = 1, 2$ are:

$$\bar{G}_0^1(s) = \begin{bmatrix} \frac{0.06346s + 1.3}{1.641s^2 + 2.382s + 1} & \frac{0.01814s + 0.3667}{5.781s^2 + 3.812s + 1} \\ \frac{0.01863s + 0.3762}{7.571s^2 + 4.458s + 1} & \frac{0.07511s + 1.544}{1.354s^2 + 2.201s + 1} \end{bmatrix} \quad (2.26)$$

$$\bar{G}_0^2(s) = \begin{bmatrix} \frac{0.06058s + 1.239}{1.582s^2 + 2.145s + 1} & 0 \\ \frac{0.01792s + 0.3362}{6.187s^2 + 4.03s + 1} & \frac{0.07027s + 1.441}{1.219s^2 + 1.871s + 1} \end{bmatrix} \quad (2.27)$$

The controller design has been performed according to the design procedure 2.1.1. Using additive uncertainty model, an upper bound for equivalent complementary sensitivity functions $T_0^i(s)$, $i = 1, 2$ has been evaluated to obtain the required minimal values of phase margins PM^i , $i = 1, 2$, for which the robust stability is guaranteed. Obtained minimal values are: $PM^1 = 54^\circ$, $PM^2 = 45^\circ$.

Nominal controllers were designed for individual equivalent subsystems using Bode design approach (Kuo and Golnaraghi, 2003) with selected required phase margins of 65° . Fulfillment of robust stability condition (2.23) for each mode is examined in Fig. 3. The closed-loop switched system stability condition (2.12)

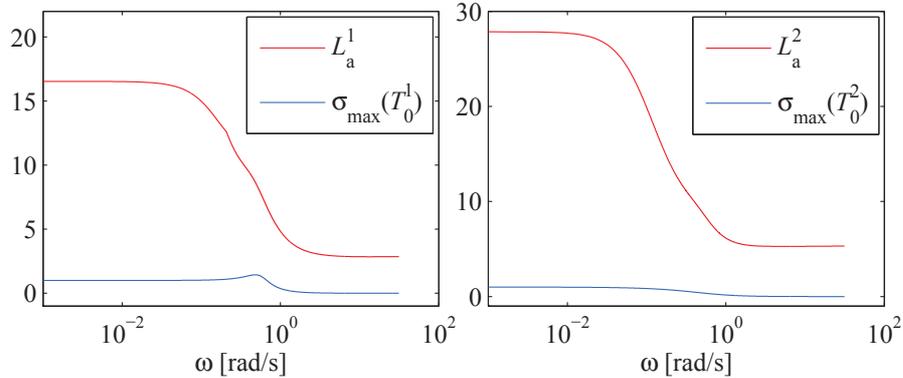


Figure 3: Verification of the robust stability condition in each mode (2.23)

has been examined in Fig. 4. The switched system is stable in all modes including the switching between individual controllers. The designed robust switched

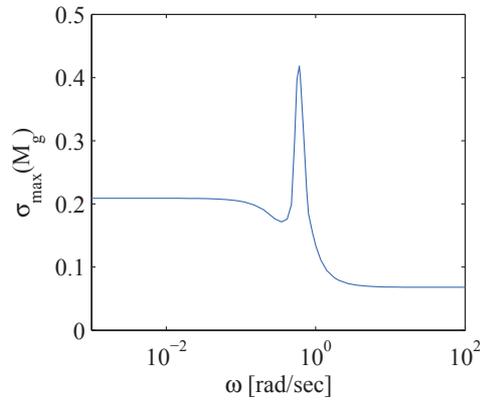


Figure 4: Verification of the switched stability condition (2.12)

controller was implemented on the real plant and tested by switching between individual operation modes (realized by connecting/disconnecting the interaction). Moreover, during experiment the loads were being changed independently within the range in both DC motors.

The results of experiments shown in Fig. 5 and Fig. 6 confirm stability and achieving desired performance under load and set-point changes in both operation modes as well as during switching between individual operation modes.

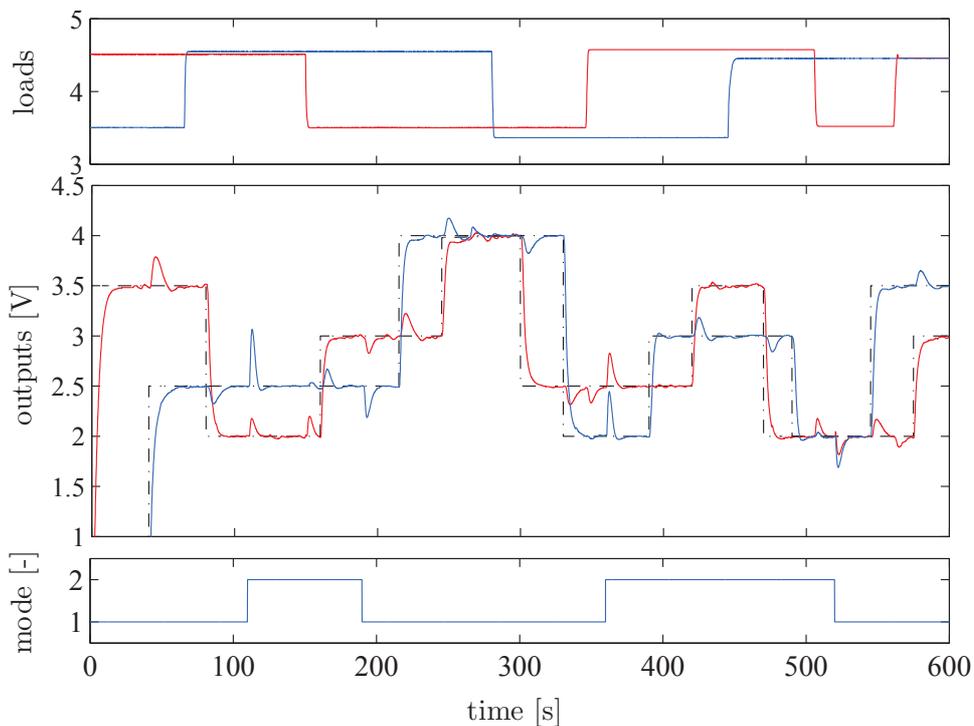


Figure 5: Time responses of measured system outputs, loads and operation mode

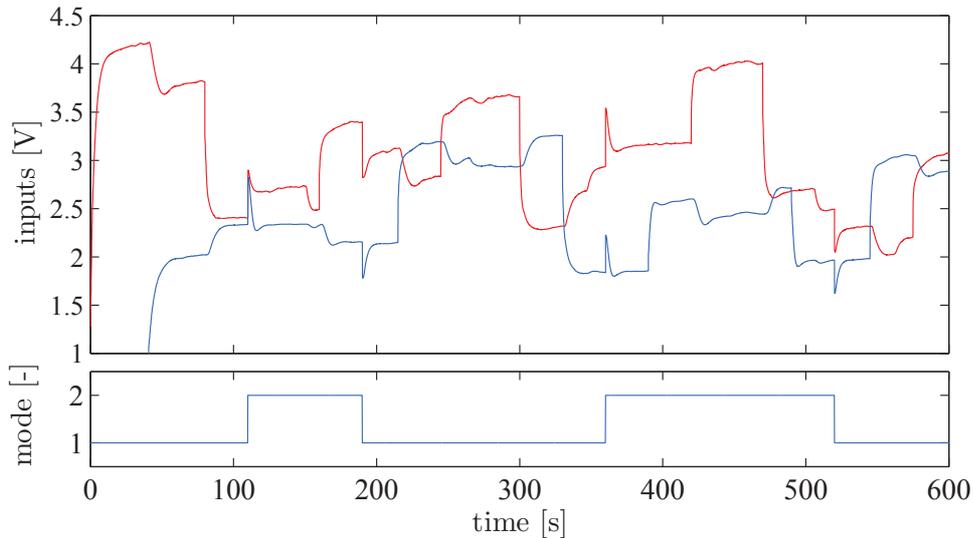


Figure 6: Time response of measured inputs and operation mode

3 Decentralized Model Predictive Control: Frequency Domain Approach

A novel hybrid control structure combining model predictive control strategy and frequency-domain strategy is introduced. The methodology for synthesis of decentralized control yields a highly effective control for complex multi-input and multi-output systems with the presence of strong couplings. The overall system is decomposed into single subsystems separated by control structure. A predictive control theory is used to formulate a set of local controllers for the control of individual subsystems. The presented DMPC scheme offers an effective solution of incorporating the non-diagonal system parts into the controller design using finite horizon objective functions to eliminate such conservatism. Stability and performance in individual subsystems are fulfilled for the full system.

Consider the discrete-time linear multivariable system (3.1) obtained from continuous-time version which is discretized with an appropriate sampling time T_s . Assume a discrete frequency response is periodic with respect to the sampling frequency $\omega_s = 2\pi/T_s$ and represented only for frequencies up to half of the sampling frequency, i.e. $\omega \in \langle 0; \omega_s/2 \rangle$.

$$y(t) = G(z)u(t) \quad (3.1)$$

For given discrete-time transfer function matrix $G(z) \in \mathbb{R}^{m \times m}$ with $z = e^{sT_s}$ it is possible to plot the frequency response either in a complex plane or in logarithmic coordinates considering $z = e^{sT_s} = e^{j\omega T_s}$ by analogy with the continuous-time case. We study the problem to find a decentralized controller

$$R(z) = \text{diag}\{R_i(z)\}_{m \times m} \quad (3.2)$$

to guarantee specified performance of the full system. Assume, that $G(z)$ can be split into diagonal and off-diagonal parts describing respective models of de-

coupled subsystems $G_d(z)$ and interactions $G_w(z)$

$$G(z) = G_d(z) + G_w(z) \quad (3.3)$$

where

$$\begin{aligned} G_d(z) &= \text{diag}\{G_i(z)\}_{m \times m}, \quad \det G_d(z) \neq 0 \\ G_w(z) &= G(z) - G_d(z) \end{aligned} \quad (3.4)$$

Controller design is performed using Equivalent Subsystems Method (Kozáková, 2012) which allows independent controller design for local equivalent subsystems generated based on the full system transfer function matrix.

3.1 Identification of Equivalent Subsystems

The generated equivalent subsystems stored in the form of vectors of real and imaginary parts depend on the frequency. From real and imaginary parts of individual equivalent subsystems the respective values of moduli and phases are obtained. Subsequently, from these values complex frequency response data models (IDFRD) are obtained using the frequency range determined by the sampling time T_s . The equivalent subsystems are identified as linear models

$$y(t) = \frac{B(z)}{F(z)}u(t) + e(t) \quad (3.5)$$

To estimate the model a cut curve fitting approach (Ljung, 2004) based on least-squares method to estimate the model was used (more details about available software tools can be found in Ljung (2008)).

Diagonal system transfer function matrix is composed of m identified equivalent subsystems obtained in the form of discrete-time transfer function models, for which a set of local SISO predictive controllers are designed.

$$G^{eq}(z) = \begin{bmatrix} G_1^{eq}(z) & 0 & \dots & 0 \\ 0 & G_2^{eq}(z) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & G_m^{eq}(z) \end{bmatrix} \quad (3.6)$$

where

$$G_i^{eq}(z) = \frac{B_i^{eq}(z^{-1})}{A_i^{eq}(z^{-1})} = \frac{b_0 + b_1 z^{-1} + \dots + b_{nb} z^{-nb}}{1 + a_1 z^{-1} + \dots + a_{na} z^{-na}} \quad (3.7)$$

for $i = 1, \dots, m$.

3.2 Decentralized Generalized Predictive Control

Consider the process with m inputs and m outputs (3.6) is described by the set of m CARIMA models (Camacho and Bordons, 2004; Levine, 1999)

$$\hat{A}_i^{eq}(z^{-1})y_i(t) = B_i^{eq}(z^{-1})\Delta u_i(t-d-1) + \xi_i(t) \quad (3.8)$$

where

$$\hat{A}_i^{eq}(z^{-1}) = \Delta A_i^{eq}(z^{-1}) \quad (3.9)$$

and

$$\begin{aligned} A_i^{eq}(z^{-1}) &= 1 + a_{1(i)}z^{-1} + \dots + a_{na(i)}z^{-na}, \\ B_i^{eq}(z^{-1}) &= b_{0(i)} + b_{1(i)}z^{-1} + \dots + b_{nb(i)}z^{-nb}, \\ C_i^{eq}(z^{-1}) &= 1 + c_{1(i)}z^{-1} + \dots + c_{nc(i)}z^{-nc}. \end{aligned} \quad (3.10)$$

$u_i(t), y_i(t)$ are the plant input and output, d is a time delay, $\xi_i(t)$ represents the effect of disturbances, and $\Delta = 1 - z^{-1}$. The integrator is introduced in the form

$$u_i(t) = \frac{1}{1 - z^{-1}} \Delta u_i(t) \quad (3.11)$$

Respective cost function is defined with the following structure

$$J_i(t) = \sum_{l=1}^{N_{y,i}} |\tilde{y}_i(t+l|t) - \tilde{w}_i(t+l|t)|_{\Pi_{y,i}}^2 + \sum_{l=1}^{N_{u,i}} |\Delta \tilde{u}_i(t+l-1|t)|_{\Pi_{u,i}}^2 \quad (3.12)$$

where Π_y, Π_u denote appropriate symmetric positive (semi)definite weighting matrices and N_y, N_u denote prediction and control horizons, respectively. The output prediction model is obtained via a set of diophantine equations (Camacho and Bordons, 2004; Rossiter, 2003). The vector form of the prediction model is as follows

$$\begin{aligned} \tilde{y}_i(t) &= G_i \tilde{u}_i(t) + \tilde{G}_i(z^{-1}) \Delta u_i(t+l-1) + \tilde{F}_i(z^{-1}) y_i(t) = \\ &= G_i \tilde{u}_i(t) + f_i \end{aligned} \quad (3.13)$$

$$\tilde{y}_i(t) = \begin{bmatrix} \tilde{y}_i(t+d+1|t) \\ \vdots \\ \tilde{y}_i(t+d+N_i|t) \end{bmatrix}, \quad \tilde{u}_i(t) = \begin{bmatrix} \Delta u_i(t) \\ \vdots \\ \Delta u_i(t+N_i-1) \end{bmatrix}, \quad (3.14)$$

$$\tilde{F}_i = \begin{bmatrix} F_{i,d+1}(z^{-1}) \\ \vdots \\ F_{i,d+N_i}(z^{-1}) \end{bmatrix}, \quad G_i = \begin{bmatrix} g_0 & 0 & \dots & 0 \\ g_1 & g_0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N_i-1} & g_{N_i-2} & \dots & g_0 \end{bmatrix}, \quad (3.15)$$

$$\tilde{G}_i(z^{-1}) = \begin{bmatrix} (G_{i,d+1}(z^{-1}) - g_0)z \\ (G_{i,d+1}(z^{-1}) - g_0 - g_1 z^{-1})z^2 \\ \vdots \\ (G_{i,d+1}(z^{-1}) - g_0 - g_1 z^{-1} - \dots - g_{N_i} z^{-N_i})z^{N_i} \end{bmatrix} \quad (3.16)$$

where f_i denotes the *free response* and the rest with the constant matrix G_i denotes the *forced response*. Resulting minimization problem is as follows:

$$\begin{aligned} J_i(t) &= \left(G_i \tilde{u}_i(t) + f_i - \tilde{w}_i(t) \right)^T \Pi_{y,i} \left(G_i \tilde{u}_i(t) + f_i - \tilde{w}_i(t) \right) + \tilde{u}_i^T \Pi_{u,i} \tilde{u}_i = \\ &= \frac{1}{2} \tilde{u}_i^T H \tilde{u}_i(t) + b_i^T \tilde{u}_i(t) + f_{0,i} \end{aligned} \quad (3.17)$$

where

$$\begin{aligned}
H_i &= 2 \left(G_i^T \Pi_{y,i} G_i + \Pi_{u,i} \right) \\
b_i^T &= 2 \left(f_i - \tilde{w}_i(t) \right)^T \Pi_{y,i} G_i \\
f_{0,i} &= \left(f_i - \tilde{w}_i(t) \right)^T \Pi_{y,i} \left(f_i - \tilde{w}_i(t) \right)
\end{aligned} \tag{3.18}$$

According to the receding horizon (Maciejowski, 2002), the minimization of $J_i(t)$ has analytical solution in the form

$$\Delta u_i(t) = K_i(\tilde{w}_i(t) - f_i) \tag{3.19}$$

Let the reference trajectory to be constant over the prediction horizon, i.e. $w(t+l) = w(t)$. Considering (3.13) the control law (3.19) can be rewritten as follows:

$$1 - z^{-1} \sum_{l=1}^{N_{y,i}} k_{i,l} G_{i,l}(z^{-1}) \Delta u_i(t) = \sum_{l=1}^{N_{y,i}} k_{i,l} (\tilde{w}_i(t+l) - \sum_{l=1}^{N_{y,i}} k_{i,l} \tilde{F}_i(z^{-1}) y_i(t)) \tag{3.20}$$

and the respective pole-placement control structure (Landau, 1998) is given as

$$R_i(z^{-1}) \Delta u(t) = T_i z^{-1} w(t) - S_i(z^{-1}) y(t) \tag{3.21}$$

where

$$R_i(z^{-1}) = 1 + z^{-1} \sum_{l=1}^{N_{y,i}} k_{i,l} G_{i,l}(z^{-1}) \tag{3.22}$$

$$S_i(z^{-1}) = \sum_{l=1}^{N_{y,i}} k_{i,l} F_{i,l}(z^{-1}) \tag{3.23}$$

$$T_i(z^{-1}) = \sum_{l=1}^{N_{y,i}} k_{i,l} \tag{3.24}$$

The controller structure is depicted in Fig. 7. When considering constraints on inputs $\langle \underline{u}_i, \bar{u}_i \rangle$, outputs $\langle \underline{y}_i, \bar{y}_i \rangle$ or rate inputs $\langle \underline{du}_i, \bar{du}_i \rangle$, the solution of the optimization problem is found using a quadratic programming solver.

Resulting quadratic programming problem for $i = 1, \dots, m$ subsystems is in the form

$$\begin{aligned}
&\underset{\Delta u_i}{\text{minimize}} && \frac{1}{2} \Delta u_i^T H_i \Delta u_i + b_i^T \Delta u_i \\
&\text{subject to} && \Theta_i \tilde{u}_i(t) \leq \mathbf{r}_i
\end{aligned} \tag{3.25}$$

where

$$\Theta_i = \begin{bmatrix} I_i \\ -I_i \\ Z_i \\ -Z_i \\ G_i \\ -G_i \end{bmatrix}, \quad \mathbf{r}_i = \begin{bmatrix} \bar{u}_i \\ -\underline{u}_i \\ \bar{du}_i - L_i u_i(t-1) \\ -\underline{du}_i + L_i u_i(t-1) \\ \bar{y}_i - f_i \\ -\underline{y}_i + f_i \end{bmatrix} \tag{3.26}$$

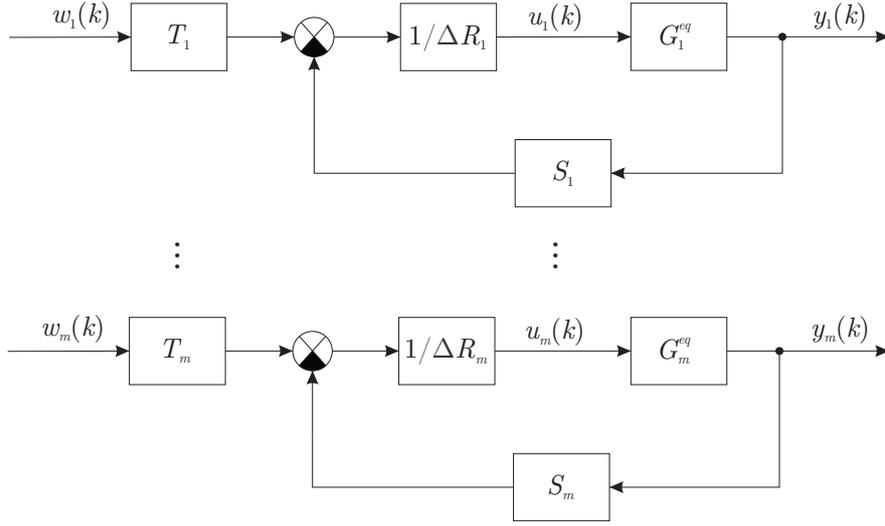


Figure 7: Polynomial structure of the decentralized controller for MIMO system represented by equivalent subsystems

and

$$Z_i = \begin{bmatrix} 1 & 0 & 0 \dots & 0 \\ 1 & 1 & 0 \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 \dots & 1 \end{bmatrix}, \quad L_i = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad (3.27)$$

3.3 Controller Design Procedure

The proposed control strategy is based on designing local predictive controllers for m individual equivalent subsystems in a such a way that by solution of m control problems ensuring required performance for equivalent subsystems, the closed-loop performance for overall system is guaranteed. The decentralized predictive control design procedure results from the above development and consists of the following steps (Kozáková and Krasňanský, 2015a):

1. Plant model discretization:

- Discretization of the continuous-time plant $G(s) \in \mathbb{R}^{m \times m}$ using an appropriately chosen sampling time T_s .
- Specification of the sampling frequency and of the feasible frequency range.

$$\omega_s = 2\pi/T_s, \quad \omega \in \langle 0; \omega_s/2 \rangle$$

2. Generating equivalent subsystems

- Partition of obtained discrete-time transfer function matrix $G(z)$ into the diagonal $G_d(z)$ and off-diagonal part $G_w(z)$ according to (3.4).
- Calculation and plotting of characteristic loci $g_i(z)$, $i = 1, \dots, m$ of the matrix $G_w(z)$ for $z = e^{j\omega T_s}$, where $\omega \in \langle 0, \omega_s/2 \rangle$.

- Choosing characteristic function $g_k(z)$ for a fixed $k \in \{1, \dots, m\}$.
- Generating and plotting discrete frequency responses of independent equivalent subsystems for selected $g_k(z)$

$$G_{ik}^{eq}(z) = G_{di}(z) + g_k(z), \quad i = 1, \dots, m; \quad k \in \{1, \dots, m\}$$

3. Identification of linear models (of appropriate order) of equivalent subsystems using the frequency responses data models obtained from obtained moduli and phases values of equivalent subsystems.
4. Independent design and tuning of m local SISO GPC controllers with specified performance requirements for all m equivalent subsystems G_i^{eq} , $i = 1, \dots, m$ using GPC design methodology described in Section (3.1).
5. Verification of closed-loop stability and achieved performance of individual feedback loops of equivalent subsystems under local predictive controllers and of the overall system respectively.

It should be noted that to verify closed-loop stability of individual subsystems any of the well-known stability conditions can be applied. Derivation of the characteristic equations of the closed-loops for equivalent subsystems can be easily obtained by substituting the analytic solutions (3.21) into the transfer function models of the system (3.8).

3.4 Example

The proposed design approach to design decentralized model predictive controller has been applied for a 3x3 MIMO system. The system is described by the transfer function matrix in the form

$$G(s) = \begin{bmatrix} \frac{2.507}{63.67s + 1.033} & \frac{1.37}{1426s + 85s + 0.933} & \frac{1.553}{1765s^2 + 90s + 1} \\ \frac{1.527}{2700s^2 + 120s + 1} & \frac{3.098}{93.67s + 1.033} & \frac{2.03}{1832s^2 + 88s + 1} \\ \frac{2.039}{2500s^2 + 163.3s + 1} & \frac{1.143}{2200s^2 + 123.3s + 1} & \frac{3.968}{76.67s + 1} \end{bmatrix} \quad (3.28)$$

This model has been discretized using a sampling time $T_s = 30$. The decentralized controller was designed according to the design procedure 3.3. The discrete-time linear models have been obtained by identification from the frequency response data (see char Fig. 8). The corresponding identified discrete-time transfer functions are in the form

$$G^{eq}(z^{-1}) = \begin{bmatrix} G_1^{eq}(z^{-1}) & 0 & 0 \\ 0 & G_2^{eq}(z^{-1}) & 0 \\ 0 & 0 & G_3^{eq}(z^{-1}) \end{bmatrix} \quad (3.29)$$

where

$$G_1^{eq}(z^{-1}) = \frac{1.366z^{-1} + 0.0111z^{-2}}{1 - 0.9069z^{-1} + 0.1513z^{-2}} \quad (3.30)$$

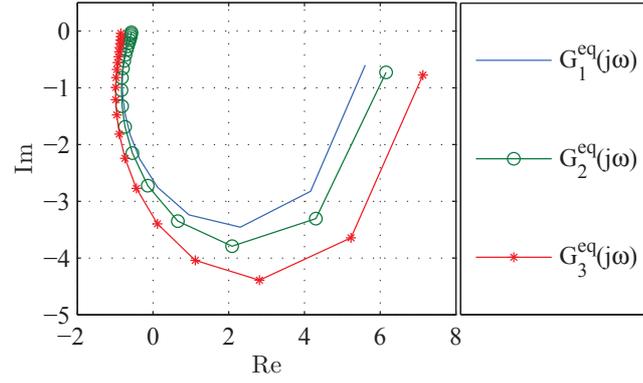


Figure 8: Nyquist frequency response of equivalent subsystems for selected $g_2(z)$

$$G_2^{eq}(z^{-1}) = \frac{1.275z^{-1} + 0.04237z^{-2}}{1 - 0.9714z^{-1} + 0.11833z^{-2}} \quad (3.31)$$

$$G_3^{eq}(z^{-1}) = \frac{1.715z^{-1} + 0.08163z^{-2}}{1 - 0.9472z^{-1} + 0.1747z^{-2}} \quad (3.32)$$

Local GPC controllers in the polynomial form (3.21) were designed using the following parameters: $N_y = 15, N_u = 15$, weights $\Pi_{y1} = 40, \Pi_{y2} = 30$ and $\Pi_{y3} = 25$.

The simulation results for closed-loop system in Fig. 9 prove that the system is stable with good control performance represented by the zero steady-state error achieved for all system output variables.

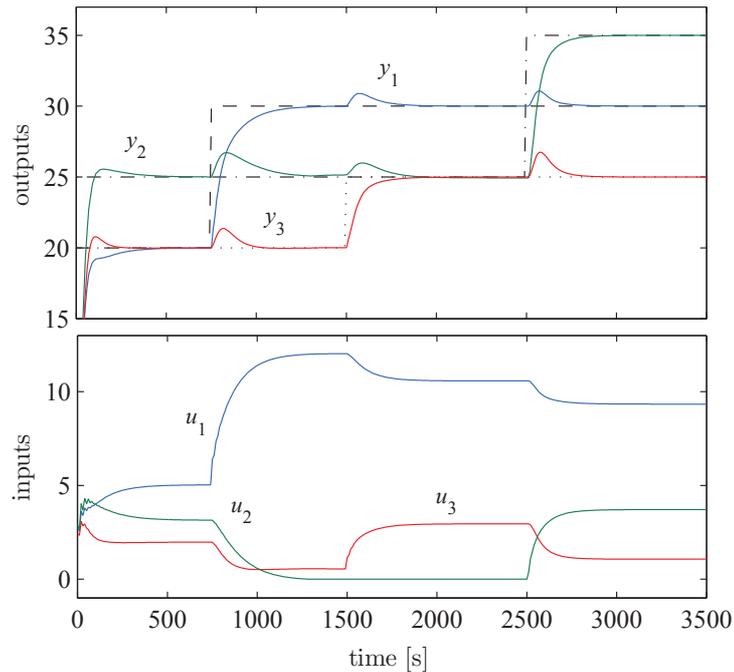


Figure 9: Time responses of the system inputs, outputs and set-points

4 Hardware Implementation of Control Algorithms

The methodology for the implementation of control algorithms using FPGAs is presented. The complexity of control algorithms increases hand in hand with higher functionality requirements and the time available to complete the calculations decreases to keep pace with ever-faster electronic devices. Therefore, microprocessor-based solutions are not longer suitable to execute the control algorithm within the accessible time limit. A potential solution to such limitations is to use of hardware-based control systems. FPGAs platforms allow the control algorithms to be implemented by programming reconfigurable hardware logic resources of the device. The control algorithm has to be manually translated into individual operations, that are subsequently encoded using the so-called Hardware Description Language (HDL) such as VHDL or Verilog and the final code is then synthesized to program a device.

Hardware implementation requires using designs based on arithmetic operations. Therefore, starting with conventional PID control algorithms we present necessary modifications of control design approaches proposed in this thesis to obtain an implementable control algorithms.

4.1 Digital PID Controller

Implementation of PID controllers using microprocessors and DSPs is well known (Tang, 2001). On the other hand, PID implementation using FPGAs is relatively new topic. The standard form of PID controller output in continuous-time has the following form

$$u(t) = k_p \left(e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_d \frac{de(t)}{dt} \right) \quad (4.1)$$

where the adjustable parameters are the proportional gain k_p , the reset time T_i and the derivative time T_d , while $u(t)$ is the control output and $e(t)$ is the error signal (set-point response level – measured response). To reduce the resources consumed for the design, a serial or parallel design can be applied. The advantage of serial approach consists in that only one adder and one multiplier are needed (Zhao et al., 2005). The parallel form of PID controller on the other hand, represents the non-interacting realization. The other parts in the implementation include datapath circuits represented by registers, multiplexers and circuits for arithmetic operations.

Hardware implementation of control algorithms requires their discrete-time representation. Thus, for selected sample time T_s , the equation (4.1) can be discretized

$$u(k) = k_p e(k) + k_i \sum_{j=0}^{k-1} e(j) + k_d (e(k) - e(k-1)) \quad (4.2)$$

where k is a discrete time instant, $k_i = k_p T_s / T_i$ is the integral coefficient and $k_d = k_p T_d / T_s$ is the derivative coefficient. More effective is to use its incremental form:

$$u(k) = u(k-1) + \Delta u(k) = u(k-1) + g_p e(k) + g_i e(k-1) + g_d e(k-2) \quad (4.3)$$

where

$$g_p = k_p + k_d$$

$$g_i = -k_p - 2k_d + k_i$$

$$g_d = k_d$$

In (4.3) the accumulation of all past errors is avoided, which is desirable in the software implementation for fast computation. Hardware implementation is based on the parallel design (Chang, Moallem, and Wang, 2004), which allows using arithmetic unit for each operation, where every unit is represented by an adder or a multiplier. Following that, the controller (4.3) is decomposed into basic arithmetic operations:

$$e(k) = \omega(k) - y(k) \quad (4.4)$$

$$\rho_p = g_p e(k) \quad (4.5)$$

$$\rho_i = g_i e(k-1) \quad (4.6)$$

$$\rho_d = g_d e(k-2) \quad (4.7)$$

$$v_1 = \rho_p + \rho_i \quad (4.8)$$

$$v_2 = \rho_d + u(k-1) \quad (4.9)$$

The calculated current control output is in the form

$$u(k) = v_1 + v_2 \quad (4.10)$$

The corresponding parallel PID design (see Fig. 10) requires a total of 2 combinational logic multipliers, 3 adders and 3 registers (Krasňanský, Dvorščák, and Kozák, 2014).

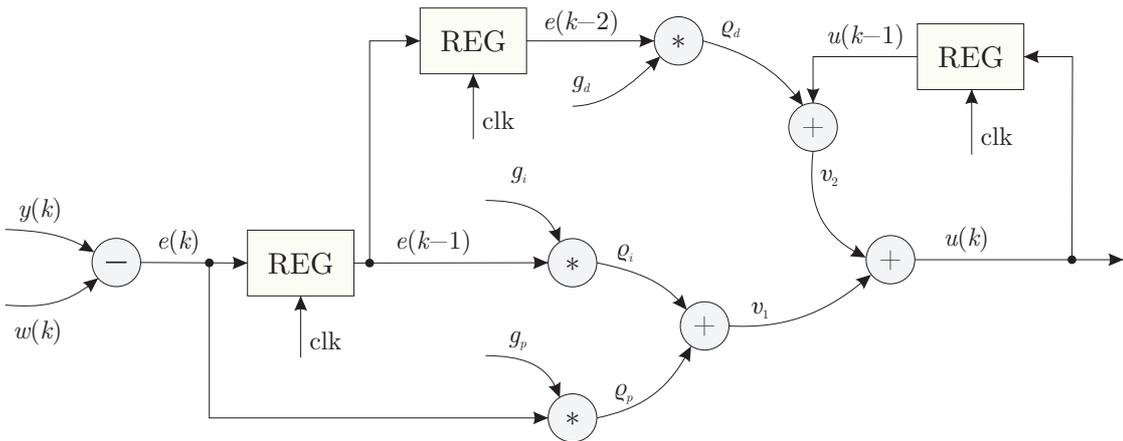


Figure 10: Parallel design of incremental PID algorithm

To control the plant a feedback control loop structure depicted in Fig. 11 is used. Each block in the scheme represents an entity in the algorithm. This

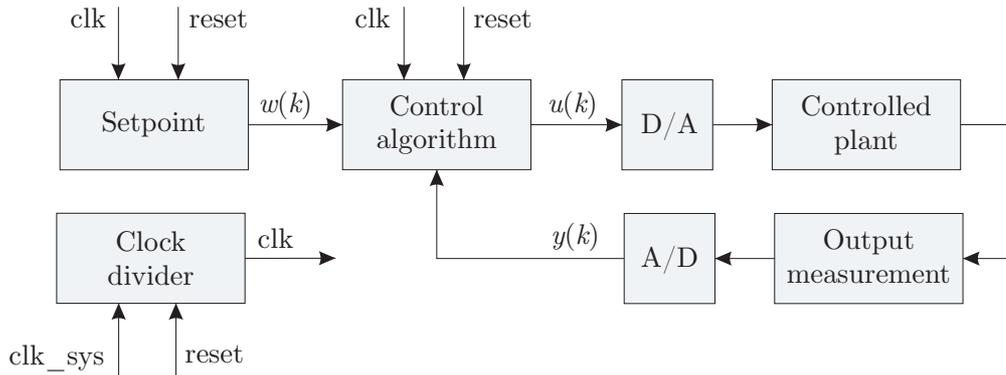


Figure 11: Control loop block scheme

structure is used in all implementation schemes of control algorithms presented in the thesis. The entity *Set-point* allows changing of the set-point value in time. The values of the reference trajectory are predefined of 12-bit size. Clock divider is used to change the clock cycle frequency in order to obtain different frequencies related to different blocks. For instance, D/A block operates at the frequency 10 MHz, but the block *Control algorithm* uses the frequency 10 Hz as the sampling time. Functionality of the entity *Control algorithm* is designed according to the selected controller structure. The control design in Fig. 10 can be described as follows. The clock signal clk is used to control the sampling frequency. In order to produce the current control error $e(k)$, binary functions of negation and subtraction are used to generate the negation of $y(k)$. Registers are used to store obtained intermediate results. On the other hand, multipliers and adders are used to perform multiplication and addition of input signals according to the specified arithmetic operations.

The bit word length and radix setting of input, output signals must be determined to ensure the fidelity of the algorithm (Krasňanský and Dvorščák, 2015). Decimal numbers are implemented using fixed-point arithmetic and bit widths of the data are determined carefully at every operation, since every addition or subtraction causes adding an extra bit. The control input signal is bounded within limits with respect to the resolution of D/A converter and to the plant and FPGA device working range. Using incremental PID algorithm, the implementation of anti-windup system is straightforward.

The control input value u_{in} is being checked and the output u_{out} is determined according to the following condition

$$u_{out} = \begin{cases} u_{max} & \text{if } u_{in} > u_{max} \\ u_{in} & \text{if } u_{max} \geq u_{in} \geq u_{min} \\ u_{min} & \text{if } u_{in} < u_{min} \end{cases} \quad (4.11)$$

where u_{in} represents the control input before saturation and u_{out} is saturated control input variable. Bounded signal is latched at register *REG*, thus becomes $u(k-1)$ of the next cycle. Moreover, in the velocity form of the algorithm, no summation appears avoiding the windup problem (Seborg, Edgar, and Mellichamp, 2003).

4.2 Switched PID Controller

Consider that in each operation mode i , a stable PI (PID) controller $R^i(s)$, $i = 1, \dots, p$ is designed and the nominal controller $R_{00}(s)$ as well as controllers $R(s)$ are calculated according to (2.19) and (2.20). Implementation on FPGA platform requires to decompose controller equation into basic arithmetic operations. At first, calculated controllers are discretized using appropriate sampling time T_s . Consequently, the parallel design structure (Fig. 10) of individual controllers in each operation mode $i = 1, \dots, p$ is obtained. The control algorithms for all p controllers are developed in VHDL based on the block scheme in Fig. 11.

The structure of the code is divided into several parts (entities), each representing one block. Switching between controllers is realized by 2-position slide switch situated on the FPGA kit which generates constant high or low inputs depending on its position. It has been programmed such that each position of the button allows just one controller to be active at the moment. A possible implementation scheme of the switch is in Fig. 12. A multiplexer allows digital signals from several sources to be routed onto a single bus or line. Input sw0 allows the source of the signal to be chosen. In case of p operation modes switching between corresponding p controllers can be performed using of $p/2$ 2-position switch buttons.

A logic 1 on the sw0 line will connect the input bus in_a1 and in_b1 to the output bus out_a and out_b, respectively. On the other hand, a logic 0 on the sw0 line will connect input bus in_a2 to output bus out_a and out_b, respectively. Calculated control input signal for each controller \bar{R}^i , $i = 1, \dots, p$ is bounded

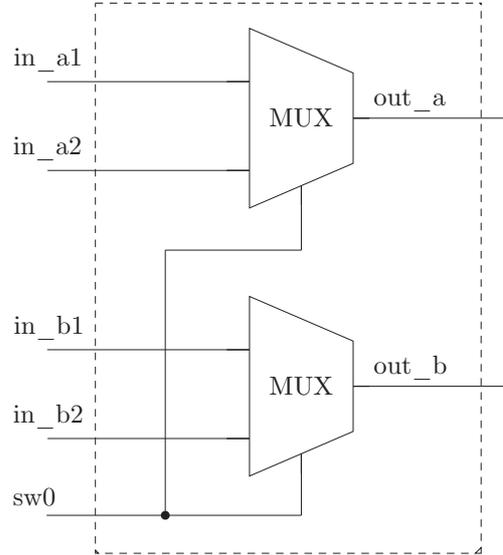


Figure 12: Switch implementation scheme

within limits with respect to the resolution of D/A converter and to the plant working range.

4.3 Decentralized Model Predictive Controller

The frequency domain predictive method is easier for application and can improve performance of control circuit. The presented predictive control method can be used in practice on real devices like FPGA without needs of complicated numerical calculations.

Consider the control law to be described by the set of RST polynomials representing local SISO predictive controllers designed for individual equivalent subsystems $G_i^{eq}(z)$, $i = 1, \dots, m$

$$R_i(z^{-1})\Delta u_i(t) = T_i(z^{-1})w_i(t) - S_i(z^{-1})y_i(t) \quad (4.12)$$

Assuming $\Delta u_i(t) = (1 - z^{-1})u_i(t)$ and after some manipulations, one obtains

$$u_i(t) = T_i(z^{-1})w_i(t) - S_i(z^{-1})y_i(t) - R_i(z^{-1}) \quad (4.13)$$

where

$$\begin{aligned} R_i(z^{-1}) &= 1 + r_1^i z^{-1} + r_2^i z^{-2} + \dots + r_a^i z^{-a} \\ S_i(z^{-1}) &= s_1^i z^{-1} + s_2^i z^{-2} + \dots + s_b^i z^{-b} \\ T_i(z^{-1}) &= \tau^i \end{aligned} \quad (4.14)$$

Hardware implementation requires decomposition of the control law (4.13) into basic arithmetic operations, where each operation is represented by an adder or multiplier:

$$\nu^i = \tau^i \omega^i(t) \quad (4.15)$$

$$\alpha_1^i = r_1^i u^i(t-1) \quad (4.16)$$

$$\vdots \quad (4.17)$$

$$\alpha_m^i = r_a^i u^i(t-a)$$

$$\gamma_0^i = s_0^i y^i(t) \quad (4.18)$$

$$\gamma_1^i = s_1^i y^i(t-1) \quad (4.19)$$

$$\vdots \quad (4.20)$$

$$\gamma_n^i = s_b^i y^i(t-b)$$

$$\mu_1^i = \gamma_1^i + \gamma_2^i + \dots + \gamma_b^i \quad (4.21)$$

$$\mu_2^i = \alpha_1^i + \alpha_2^i + \dots + \alpha_a^i \quad (4.22)$$

$$\varepsilon^i = \nu^i - \mu_1^i \quad (4.23)$$

The calculated current control output is given as

$$u^i(t) = \varepsilon^i - \mu_2^i \quad (4.24)$$

The respective controller realization scheme representing the k^{th} polynomial controller is in Fig. 13. The control algorithm is developed using VHDL and the control scheme is similar to the principled block scheme (Fig. 11).

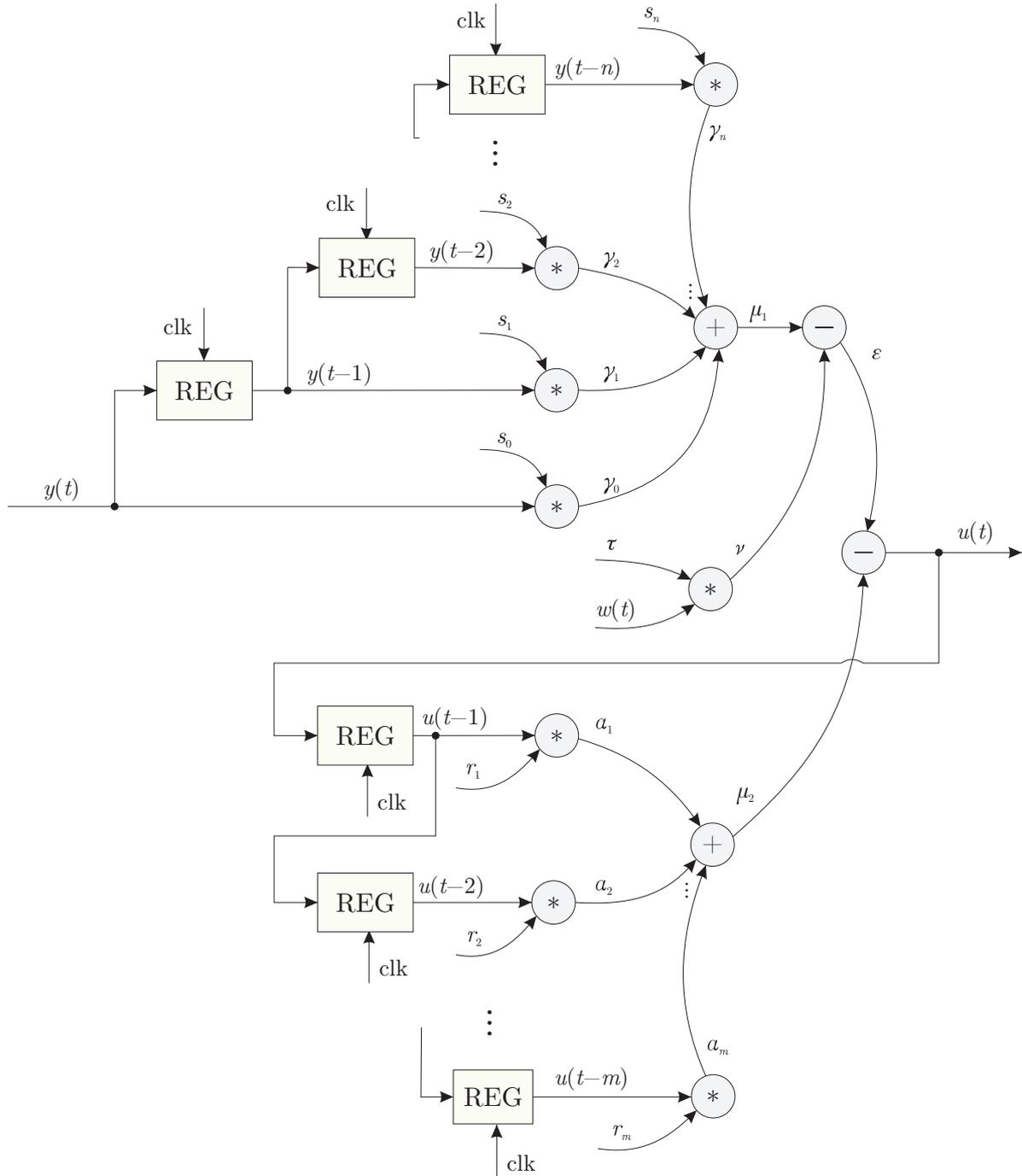


Figure 13: Polynomial GPC implementation scheme

4.4 Example

The effectiveness of two presented controller design methods, namely a switched PID control algorithm and a decentralized GPC controller has been tested experimentally on the laboratory MIMO plant with two inputs and two outputs. The plant can operate in two modes. The mathematical model of the system has been obtained by identification from the I/O data measurements in selected

working point carried out by using of two analog 2-channel oscilloscopes from Digilent Inc. Obtained transfer function models:

Operation mode 1:

$$\bar{G}^1(s) = \begin{bmatrix} \frac{0.06708z^{-1} - 0.04223z^{-2}}{1 - 1.611z^{-1} + 0.628z^{-2}} & \frac{0.007201z^{-1} - 0.01945z^{-2}}{1 - 1.765z^{-1} + 0.7821z^{-2}} \\ \frac{-0.01023z^{-1} + 0.009237z^{-2}}{1 - 1.896z^{-1} + 0.8988z^{-2}} & \frac{0.07587z^{-1} + 0.01499z^{-2}}{1 - 1.141z^{-1} + 0.2182z^{-2}} \end{bmatrix} \quad (4.25)$$

Operation mode 2:

$$\bar{G}^2(s) = \begin{bmatrix} \frac{0.04591z^{-1} - 0.003486z^{-2}}{1 - 1.618z^{-1} + 0.6549z^{-2}} & 0 \\ \frac{0.005095z^{-1} - 0.01111z^{-2}}{1 - 1.779z^{-1} + 0.7969z^{-2}} & \frac{0.123z^{-1} + 0.02891z^{-2}}{1 - 1.165z^{-1} + 0.3233z^{-2}} \end{bmatrix} \quad (4.26)$$

Frequency-domain switched controller design

Local SISO controllers $\bar{R}^i(s)$, $i = 1, 2$ were designed according to design procedure 2.1 for individual equivalent subsystems using Bode design method (Kuo and Golnaraghi, 2003). To achieve overshoot-free closed-loop responses, the required phase margins of 65° have been selected. Consequently, the controllers have been discretized with sampling time $T_s = 0.1$.

The closed-loop switched system stability condition (2.12) has been examined in Fig. 14. The switched system is stable in all operation modes including the switching between individual controllers. The control algorithm was developed

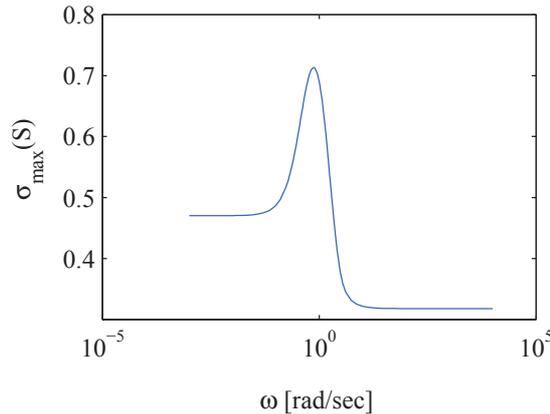


Figure 14: Fulfillment of the switched stability condition (2.12)

using VHDL language and Xilinx Vivado software environment according to the control implementation structures in Fig. 11 and Fig. 10. Considering a MIMO system operating in two modes, four controller blocks were used in the scheme. Synthesized code has been implemented into Nexys-4 board.

Experimental results

The real system measurements depicted in Fig. 15 and Fig. 16 demonstrate the effectiveness and good performance of the FPGA-based switched decentralized controller in individual operation modes as well as during switching between them.

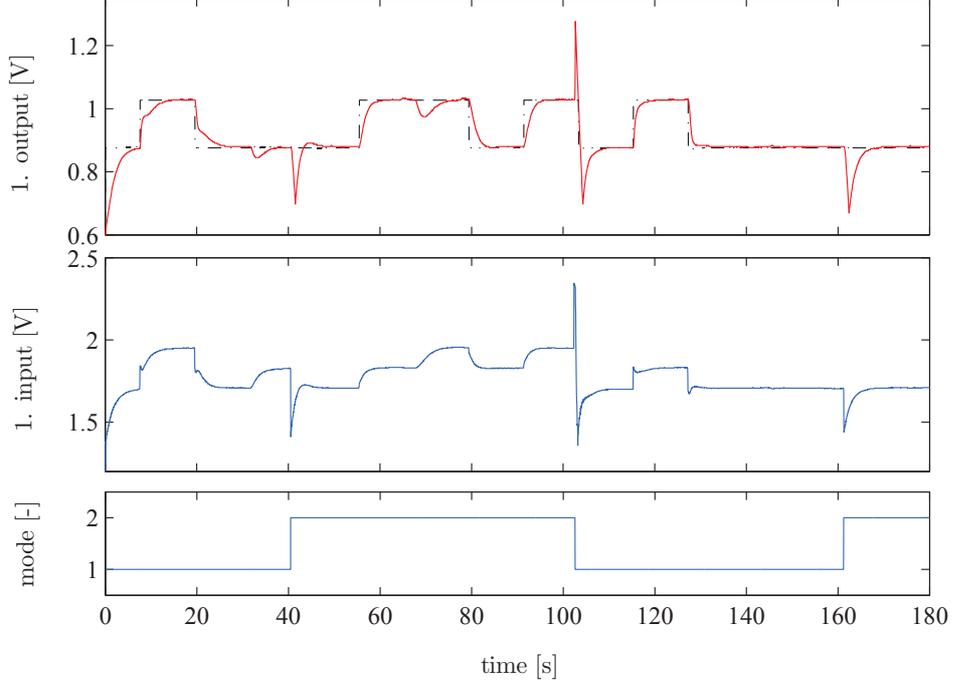


Figure 15: Measured reference, first input and first output signal during experiment

Decentralized predictive controller design

Consider the system operates only in the first mode. Using the control design procedure 3.3 the discrete-time linear models have been obtained by identification as fourth-order Output-Error models with sampling time $T_s = 0.1$

$$G_1^{eq}(z^{-1}) = \frac{0.04845z^{-1} - 0.04163z^{-2} - 0.03074z^{-3} + 0.02632z^{-4}}{1 - 3.122z^{-1} + 3.615z^{-2} - 1.835z^{-3} + 0.3434z^{-4}} \quad (4.27)$$

$$G_2^{eq}(z^{-1}) = \frac{0.1564z^{-1} - 0.1982z^{-2} + 0.009087z^{-3} + 0.03974z^{-4}}{1 - 2.564z^{-1} + 2.234z^{-2} - 0.6963z^{-3} + 0.0306z^{-4}} \quad (4.28)$$

Local GPC controllers in the polynomial form (3.21) have been designed with the following design parameters: $N_y = N_u = 6$, weights $\Pi_{y1} = 40$, $\Pi_{y2} = 20$ and sampling time $T_s = 0.1$. The corresponding control algorithm has been developed using VHDL language and Xilinx Vivado software environment. The plant has been controlled with Xilinx Nexys-4 FPGA clocked at 100 MHz.

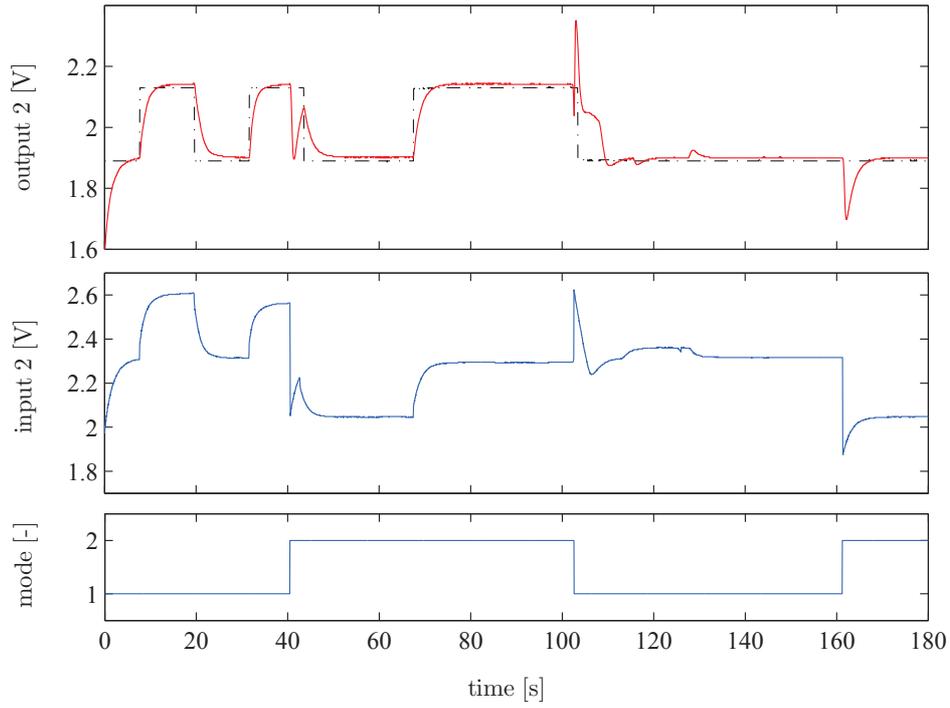


Figure 16: Measured reference, first input and first output signal during experiment

Experimental results

The results of experiments performed under the decentralized predictive controller in Fig. 17 and Fig. 18 confirm sufficient implementation as well as stability and achieving desired performance under time-varying reference trajectory.

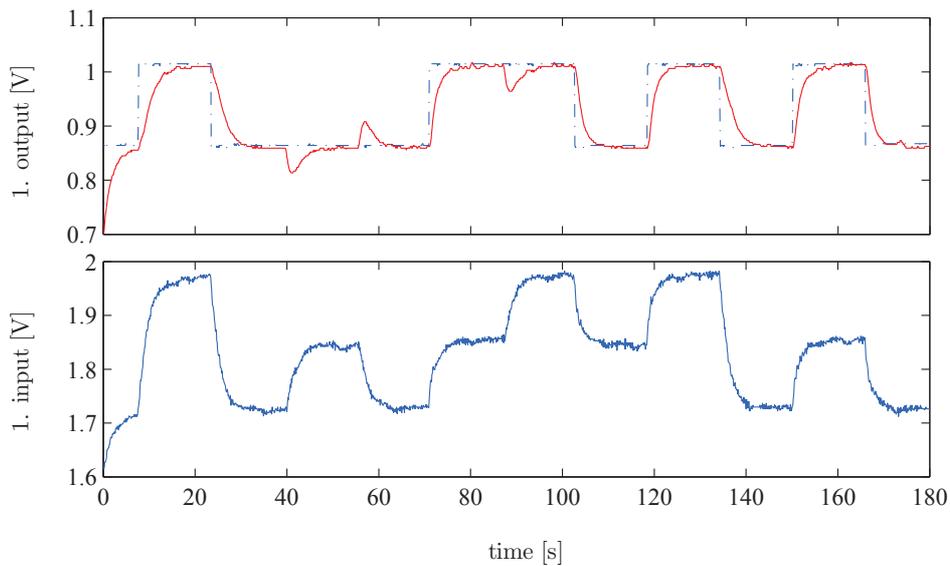


Figure 17: Measured reference, first input and first output signal during experiment

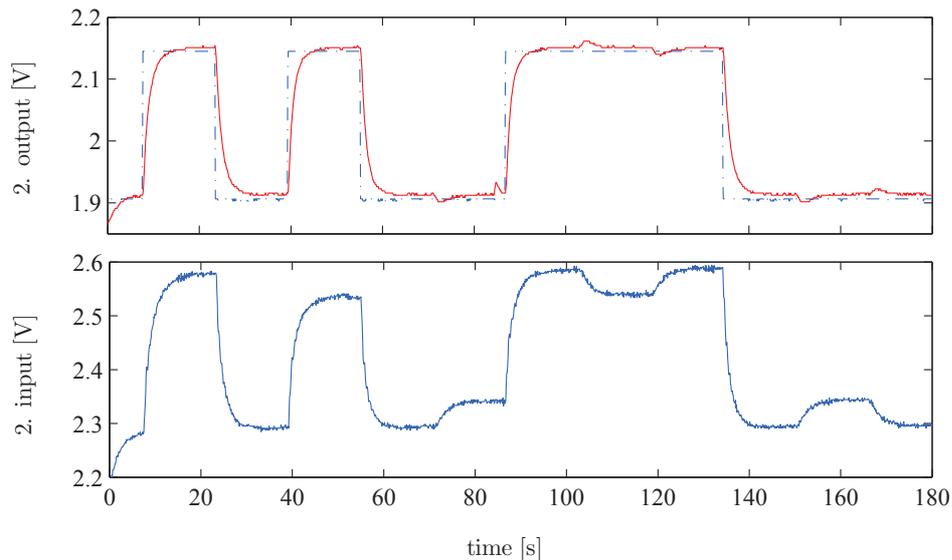


Figure 18: Measured reference, second input and second output signal during experiment

5 Conclusions

The PhD thesis deals with development of hybrid control structures for complex mechatronic systems and algorithmization of proposed control design methods for hardware implementation on FPGA platform. This study has been motivated by recent results in robust and decentralized control of complex systems including hybrid systems in the time and frequency domains. The main objective was to elaborate on the achieved theoretical results a practice-oriented design strategy and to develop new design procedures as well as to verify theoretical results on selected case studies. Novel control design approaches for single-input and single-output as well as multi-input and multi-output dynamic systems are based on the theoretical findings derived in the frequency domain.

The objectives of the thesis have been specified on the basis of the present developments in decentralized control of multivariable systems and switched linear systems control from the viewpoint of time domain and frequency domain approaches. A new control design procedure for switched linear systems developed in the frequency domain is supposed to be better to understand in the engineering community, thus its practical ability is supposed. A new approach to decentralized model predictive control design for multivariable systems developed in the frequency domain allows decompose complex control problem to several subproblems which may be solved independently with individual performance requirements. From the practical point of view a design methodology to the hardware implementation of the proposed control design methods on FPGA devices is proposed. The effectiveness of developed control algorithms is verified on several case studies. The controller design based on FPGAs is becoming more and more popular in industries, and also due to incontestable advantages, FPGA controllers are expected to be employed in production lines in the very near future.

The achieved results of the thesis are both in theoretical and practical domains. The following section summarize the main contributions according to formulated thesis objectives.

5.1 Contributions of the Thesis

5.1.1 Theoretical contributions

Theoretical contributions are in design and development of hybrid control structures and algorithms. In particular:

- **Development of the frequency-domain switched stability condition**

A new frequency-domain switched system stability condition has been derived based on the $M-\Delta$ structure of closed-loop system and the small gain theory while it uses the affine model of the system and the controller. Thus, it provides important results for the closed-loop stability of the multi-model plant in all operation modes as well as stability during switching between individual modes. The proposed method represents a sufficient stability condition for SISO and MIMO switched linear systems under arbitrary switching.

- **Development of the control design procedures in the frequency-domain**

A new frequency-domain controller design procedure for switched systems is proposed and realized in Section 2.1. The presented design approach is applicable for both SISO and MIMO multi-model systems represented in the affine form. Controller design can be performed using any frequency-domain design method. For MIMO multi-model systems, a decentralized controller is designed using Equivalent Subsystems Method, which allows independent design of local SISO controllers for individual equivalent subsystems with prescribed performance requirements. Effectiveness of the proposed design methodology has been illustrated by example of a SISO system controlled under very fast switching.

- **Development of the control design procedure guaranteeing robust stability and required nominal performance**

An extension of the frequency-domain control design procedure for robust stability in individual operation modes is proposed. An unstructured uncertainty model is used for nominal model calculation and the robust stability condition is included in the controller design. Resulting robust controller design procedure employs the switched stability condition and guarantees robust stability in individual operation modes. Moreover, a comparison of the effectiveness of developed controller design procedure and the state-space robust design approaches is demonstrated on several examples via simulations and experiments. Robust control design methods for switched systems in the time domain often lead to BMIs, thus to find a feasible solution may represent a problem. On the other hand, the

frequency-domain approaches are better understood in engineering community and therefore more eligible. Based on the simulation and experimental results of examples, the presented control design approach is believed to represent the alternative in robust (decentralized) control of switched systems, which can bring useful results.

- **Design and development of frequency-domain decentralized model predictive control for complex systems with multiple inputs and outputs**

A novel approach to the decentralized model predictive controller design in the frequency domain has been proposed in Chapter 3. It provides the extension of the applicability of equivalent subsystems approach in the field of model predictive control. The main advantage of this approach is a diagonalization of the original plant by generating a diagonal matrix of equivalent subsystems. Subsequently, from the frequency response models of the equivalent subsystems, the discrete-time linear models are identified. This way, local predictive controllers for individual equivalent subsystems can be designed and tuned with corresponding performance specifications whereas achieved stability and control performance of equivalent closed-loops are guaranteed for the full system. Important points in the design procedure are model identification from frequency responses of equivalent subsystems, and stability analysis based on polynomial control structure of the unconstrained control algorithm. The effectiveness of the proposed methodology has been demonstrated on simulations and experiments on multivariable processes.

5.1.2 Practical contributions

- **Algorithmization of developed control design approaches for hardware implementation on FPGA platforms**

Based on the proposed control design approaches, synthesizable control schemes for hardware realization have been derived and corresponding control algorithms have been developed using VHDL. In particular, control algorithms for conventional PI/PID controllers, a decentralized switched PI controller, and a decentralized GPC controller have been successfully synthesized using Xilinx design tools (ISE Design Suite and Vivado Design Suite).

- **Hardware realization and implementation of developed control algorithms**

Developed control algorithms have been implemented on FPGA boards from Xilinx: Spartan-6 SP601 and latter Nexys-4 kit. After VHDL code synthesis a bit-stream has been generated and successfully implemented on FPGAs.

- **Real-time experiments**

The applicability and effectiveness of developed FPGA-based control algorithms have been verified by real-time experiments on two mechatronic

plants. In particular, conventional PI(PID) algorithms have been designed for SISO model of a laboratory DC motor system and implemented on Spartan-6 FPGA. Decentralized control algorithms have been implemented on Nexys-4 FPGA board and tested in control of laboratory DC motors system with multiple inputs and outputs, which can operate in two modes. Experimental results of practical examples show the effectiveness and easy implementation of the control schemes providing important results for the future applications.

The effectiveness of the presented control methods have been experimentally verified by simulations and in real systems control. Achieved results were published at important international conferences and journals.

5.2 Future research perspectives

In this section we propose several possible directions and ideas that might be interesting for the future research.

The switched system stability condition presented in Chapter 2 represents only sufficient condition. Therefore, it would be interesting to find the possibilities of reducing its conservatism. The developed procedure based on this condition has been used in robust control design. In the future it would be interesting to derive a robust switched stability condition guaranteeing robust stability also during switching between individual operation modes.

The future development could deal with guaranteeing desired performance of the full system by systematic tuning of the GPC design parameters (prediction horizons, weights), as well as with the problem of guaranteeing stability under the decentralized GPC with constraints.

The results of the research in hardware implementation of control algorithms can be considered as preliminary but a relevant starting point which will prompt further interest in controller implementation. The further research could be focused on finding solutions for implementation of complex optimal controllers based on simple, easy-to-implement computation principles. In case of MPC with constraints on system variables, it would be interesting to study simple but effective algorithms solving the real-time optimization problem with less computational effort. This leads to complex matrix computations, where specific substitutions are needed to be used.

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Resumé

Predkladaná dizertačná práca sa zaoberá vývojom hybridných riadiacich štruktúr pre mechatronické systémy vo frekvenčnej oblasti ako i možnosťami hardvérovej implementácie odvodených algoritmov riadenia na FPGA štruktúrach pre potreby riadenia dynamických procesov s rýchlou dynamikou.

Cieľom teoretickej časti práce bolo rozpracovať nové prístupy k riadeniu tak jednorozmerných ako aj mnohorozmerných zložitých dynamických systémov vychádzajúce z teoretických poznatkov odvodených vo frekvenčnej oblasti. Prvý prístup k návrhu riadenia je odpoveďou na nedostatok metód a prístupov k riadeniu hybridných systémov (systémov s prepínaním) vo frekvenčnej oblasti. Zabezpečenie stability uzavretého regulačného obvodu pre systémy s prepínaním predstavuje neľahkú úlohu. Navrhnutý prístup je založený na metodike teórie malého zosilnenia a $M - \Delta$ štruktúre, ktorá rieši postačujúcu podmienku stability. V práci je odvodená frekvenčná podmienka pre stabilitu uzavretého regulačného obvodu pre systémy s prepínaním, zabezpečujúca stabilitu v jednotlivých režimoch ako i počas prepínania medzi nimi. Podmienka stability je založená na afinnom opise systému a regulátora. Na základe tejto podmienky bol v práci odvodený postup návrhu regulátora pre systémy s prepínaním, aplikovateľný tak pre SISO ako aj MIMO systémy. Odvodený postup umožňuje návrh jednoduchých PI/PID regulátorov, pričom pre výpočet parametrov regulátora je možné použiť ľubovoľnú frekvenčnú metódu.

Pre systémy s viacerými vstupmi a výstupmi bol v práci odvodený postup návrhu decentralizovaných regulátorov založený na metóde ekvivalentných pod-systémov, ktorá sa vyvíja na školiacom pracovisku posledné desaťročie a umožňuje nezávislý návrh regulátorov pre tzv. ekvivalentné podsystémy. Pre riadenie systémov s neurčitostami bol odvodený postup návrhu robustných regulátorov s prepínaním, pričom pre mnohorozmerné systémy je opäť využitá metóda ESM. Pre výpočet nominálneho modelu systému a návrh regulátora boli použité modely s dynamickou (neštruktúrovanou) neurčitostou. Robustné regulátory v afinnom tvare navrhnuté týmto postupom zabezpečujú robustnú stabilitu v jednotlivých pracovných režimoch plného systému, pričom vďaka podmienke stability garantujú stabilitu pri prepínaní medzi jednotlivými režimami. Navrhnuté metódy a prístupy boli overené simulačne na niekoľkých laboratórnych modeloch reálnych procesov ako i experimentálne riadením reálnych procesov (systém prepojených motorčekov v SISO a MIMO zapojení). Efektívnosť a kvalita navrhnutých prístupov boli porovnané s prístupmi odvodenými v časovej oblasti prostredníctvom porovnávacích simulačných príkladov. Stabilita systému s navrhnutými regulátormi bola testovaná na príkladoch i pri veľmi rýchlom prepínaní medzi režimami, pričom získané výsledky potvrdzujú teoretické predpoklady a dokazujú praktickú využiteľnosť odvodených metód a postupov. Funkcionalita tohto prístupu bola overená na príklade riadenia silno nelineárneho systému (bubnový kotol), pričom bola porovnaná kvalita riadenia pri použití jedného robustného regulátora pre celú oblasť a riadenia s uvažovaním prepínania medzi viacerými robustnými regulátormi. Na základe dosiahnutých výsledkov je možné povedať že prístupy odvodené vo frekvenčnej oblasti sú ľahšie pochopiteľné a

prakticky využiteľné v inžinierskej praxi.

Druhý prístup predstavujúci rozšírenie použiteľnosti metódy ESM v praxi je opísaný v piatej kapitole. Umožňuje návrh decentralizovaného prediktívneho riadenia pre zložité systémy s viacerými vstupmi a výstupmi. Jednou z motivácií pre odvodenie tohoto postupu bolo práve rozšírenie aplikovateľnosti ESM metódy, ktorá doteraz umožňovala použiť pre výpočet parametrov regulátora iba frekvenčné metódy. Ďalšou motiváciou bolo redukovať problémy spojené s návrhom centralizovaného regulátora pre úplný systém. Hlavnou výhodou navrhnutého prístupu je rozdelenie úlohy návrhu regulátora pre úplný systém na niekoľko jednoduchých úloh návrhu lokálnych regulátorov pre jednotlivé ekvivalentné podsystémy predstavujúce samostatné uzavreté regulačné slučky. Odvozená metóda umožňuje nezávislý návrh lokálnych regulátorov s možnosťou špecifikovať požiadavky na kvalitu pre každý regulačný obvod konkrétneho podsystému. Navyše, v prípade ak je zabezpečená stabilita jednotlivých ekvivalentných regulačných slučiek, je zároveň garantovaná i stabilita úplného systému. Postup návrhu regulátora je založený na identifikácii ekvivalentných podsystémov z frekvenčných charakteristík a získaní lineárnych modelov v tvare diskretných prenosových funkcií vhodných pre návrh prediktívnych regulátorov. Pre získané modely ekvivalentných podsystémov je možné navrhnuť lokálne SISO prediktívne regulátory v polynomiálnom tvare (GPC), ktoré umožňujú jednoducho vykonať analýzu stability uzavretého regulačného obvodu jednotlivých podsystémov. Jednou z výhod použitia GPC regulátorov pre mnohorozmerné systémy oproti iným prediktívnym metódam založeným na stavovom opise riadeného systému je práve vstupno-výstupný opis systému. Výsledkom návrhu je, že stabilita a kvalita plného systému sú garantované ak je zabezpečená stabilita a dosiahnutá kvalita v rámci jednotlivých ekvivalentných podsystémov. To umožňuje pri návrhu špecifikovať kvalitu riadenia nezávisle pre jednotlivé výstupy systému. Nový prístup pre návrh decentralizovaného prediktívneho regulátora bol overený na simulačných príkladoch (3x3 MIMO systém) i experimentálne (2x2 MIMO systém prepojených motorčekov). Dosiahnuté výsledky prispievajú k rozšíreniu aplikačných oblastí metódy ESM a prinášajú nové poznatky a možnosti pre budúci výskum v danej oblasti.

Využitím navrhnutých prístupov riadenia v kapitolách 2 a 3 je možné riadiť zložité mnohorozmerné mechatronické systémy a hybridné systémy použitím štruktúr riadenia založených na návrhu lokálnych SISO regulátorov zabezpečujúcich želanú kvalitu riadenia pre úplný systém. Rozloženie zložitých problémov riadenia na množstvo jednoduchších úloh umožňuje jednoduchú hardvérovú implementáciu algoritmov riadenia pre zložité systémy pri nižších časových a energetických nákladoch. Vďaka týmto výhodám je možné nájsť využiteľnosť odvodených štruktúr riadenia vo všetkých oblastiach mechatronických aplikácií.

Hlavným cieľom praktickej časti bolo navrhnuť štruktúry riadenia vhodné pre hardvérovú implementáciu vychádzajúc z odvodených hybridných riadiacich štruktúr pre SISO aj MIMO systémy. Použitie FPGA obvodov bolo v minulých rokoch obmedzené, nakoľko neumožňovali vytvoriť veľmi zložité zapojenia, ktoré sú v dnešných moderných konštrukciách z oblasti telekomunikácií a digitálneho spracovania signálov nutné. Pokrok v technológii výroby obvodov FPGA a

samozrejme aj vo vývoji potrebného návrhového softvéru umožnil ich použitie aj pre návrh digitálnych zapojení so zložitostou stoviek tisícov až miliónov logických hradiel.

Prvá časť šiestej kapitoly pojednáva o FPGA obvodoch z pohľadu architektúry, princípu činnosti, hlavných výhod, nevýhod a z pohľadu možností implementácie riadiacich algoritmov v porovnaní s realizáciou na mikroprocesoroch. V druhej časti kapitoly sú opísané vlastné návrhy a postupy nutné pre hardvérovú realizáciu vybraných riadiacich algoritmov na FPGA obvodoch. Algoritmy riadenia boli odvodené na základe prístupov opísaných v kapitolách 2 a 3. Ich funkčnosť bola následne experimentálne otestovaná na vybraných reálnych procesoch realizovaných na systéme prepojených motorčekov v zapojení pre SISO a MIMO systém. Matematické modely boli získané identifikáciou na základe vstupno-výstupných dát vo zvolených pracovných bodoch. Navrhnuté algoritmy riadenia boli úspešne implementované na FPGA obvodoch a zároveň bola overená funkcionálna hardvérovej realizácie. Konkrétne sa jednalo o dva typy obvodov od spoločnosti Xilinx a to vývojové dosky Spartan-6 s podporou softvéru Xilinx ISE Design Suite a Nexys-4 s podporou softvéru Xilinx Vivado Design Suite. Dosiagnuté výsledky potvrdzujú správnu funkcionálnu hardvérovej realizácie, praktickú použiteľnosť a tiež stabilitu uzavretého regulačného obvodu vo všetkých pracovných bodoch ako aj pri prepínaní medzi jednotlivými režimami (pri riadení systému s prepínaním) a kvalitu riadenia (riadenie s nulovou alebo minimálnou regulačnou odchýlkou) odvodených riadiacich algoritmov.

Na základe prehľadu súčasného stavu je možné povedať, že vývoj metód implementácie riadiacich algoritmov na FPGA štruktúrach je stále vo svojich začiatkoch. Samotná architektúra týchto obvodov nepripúšťa existenciu konkrétnej metodiky návrhu algoritmov na tieto zariadenia, čo prináša vysokú flexibilitu vo vývoji nových metód pre implementáciu. Návrh algoritmov riadenia založených na FPGA štruktúrach sa stáva viac a viac populárny v priemysle aj vďaka nepochybniteľným výhodám akými sú paralelizmus alebo rýchlosť, pričom sa očakáva že v blízkej budúcnosti budú tieto obvody zaradené vo všetkých oblastiach priemyselnej produkcie.

Napriek získaným výsledkom je možné nájsť viacero otvorených problémov, ktoré by bolo vhodné rozpracovať v ďalšom výskume. V oblasti riadenia systémov s prepínaním by bolo vhodné nájsť možnosti zníženia konzervatívnej odvodenej frekvenčnej podmienky stability, ktorá zatiaľ predstavuje postačujúcu podmienku. Samotný návrh riadenia spočíva v návrhu robustných regulátorov pre jednotlivé režimy systému, pričom je zabezpečená stabilita pri prepínaní medzi jednotlivými režimami. V ďalšom výskume by mohlo byť prínosom modifikovať podmienku stability pri prepínaní tak, aby boli zohľadnené i podmienky robustnej stability. V oblasti decentralizovaného prediktívneho riadenia ostáva otvoreným problémom zabezpečenie stability uzavretého regulačného obvodu pri uvažovaní ohraňovania na vstupné a výstupné premenné systému. Ďalší rozvoj v oblasti implementácie riadiacich algoritmov na FPGA štruktúrach by mohol byť zameraný na riešenie problémov implementácie zložitých optimálnych regulátorov (MPC s ohraňovaniami) založených na hľadaní jednoduchých výpočtových princípov, ktoré sú časovo menej náročné.