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Dissertation Thesis Abstract

Torso Electrode Significance Evaluation for Solving the Inverse Problem of Electrocardiography with a Single Dipole Cardiac Source

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ABSTRACT

This dissertation thesis focuses on the solution of the inverse problem of electrocardiography that is used to determine the origin of ectopic cardiac activity, such as premature ventricular contractions (PVCs). The inverse problem has not yet achieved widespread clinical adoption due to the impracticality of using tens to hundreds of torso electrodes to record the electrical activity of the heart. Therefore, this dissertation thesis investigates the positions of the most significant torso electrodes and explores the potential of using a reduced set of these electrodes for the solution of the inverse problem assuming a single dipole cardiac source.

The EP Solutions dataset (8 patient cases recorded during ventricular pacing) and the Bratislava dataset (13 patients with spontaneous PVCs) were used to achieve four main objectives. First, we proposed a novel methodology to determine electrode significance by analyzing the transfer matrix computed for a specific cardiac source position and using a greedy algorithm. Second, the positions of significant electrodes in both homogeneous and inhomogeneous volume conductors and for different positions of the cardiac source were investigated, and the concept of optimal lead placement was explored. Third, the impact of significant electrodes on the inverse solution accuracy, measured by localization error (LE), was studied. The LE was computed as the Euclidean distance between the inverse solution and the true origin of cardiac activity. Lastly, we introduced a two-step inverse solution to address unknown cardiac source position in clinical settings.

Our findings reveal that the positions of significant electrodes depend on the cardiac source location, with negligible differences between homogeneous and inhomogeneous torso models. Using the 32 and 64 most significant electrodes yield comparable inverse solution accuracy (LE 26.4 $\pm$ 9.9 mm and 27.3 $\pm$ 5.6 mm) to using all electrodes (LE 24.7 $\pm$ 3.7 mm). The concept of optimal lead placement was explored, showing that strategically positioned 64 electrodes can yield accurate results of the inverse solution (LE 27.0 $\pm$ 14.3 mm), even outperforming the full set (LE 28.8 $\pm$ 11.9 mm). A two-step inverse solution demonstrated its ability to provide accurate results using 32 electrodes (LE 28.8 $\pm$ 14.5 mm) compared to the full set (LE 28.8 $\pm$ 11.9 mm) with the potential to assess the reliability of the first inverse solution.

These results underscore the potential of using a reduced number of significant electrodes to solve the inverse problem, thereby facilitating the implementation of more practical configurations in clinical settings.

ABSTRAKT

Táto dizertačná práca sa zameriava na riešenie inverzného problému elektrokardiografie, ktorý sa používa na určenie miesta vzniku ektopickej srdcovej aktivity, ako sú predčasné komorové kontrakcie (PVC). Riešenie inverznej úlohy zatiaľ nemá rozsiahle klinické využitie v dôsledku nepraktickosti použitia desiatok až stoviek hrudníkových elektród na zaznamenávanie elektrickej aktivity srdca. V tejto dizertačnej práci hľadáme umiestnenie najvýznamnejších elektród na hrudníku a analyzujeme potenciál využitia redukovanej sady významných elektród na riešenie inverznej úlohy pomocou jednoduchého dipólu.

Na dosiahnutie štyroch hlavných cieľov sme použili EP Solutions data (8 súborov dát zaznamenaných počas komorovej stimulácie) a Bratislava data (13 pacientov s spontánnymi PVC). Ako prvé sme navrhli novú metódu na určenie významu jednotlivých elektród analýzou prenosovej matice vypočítanej pre konkrétnu polohu srdcového zdroja s použitím tzv. greedy algoritmu. Ďalej sme skúmali polohu významných elektród pre rôzne polohy srdcového zdroja a pre homogénne aj nehomogénne objemové vodiče hrudníka a zaviedli sme koncept optimálneho umiestnenia elektród na hrudníku. Následne sme hodnotili vplyv významných elektród na presnosť inverzného riešenia. Presnosť riešenia sme hodnotili pomocou chyby lokalizácie (LE), ktorá bola vypočítaná ako euklidovská vzdialenosť medzi inverzným riešením a skutočným miestom vzniku srdcovej aktivity. Nakoniec sme predstavili dvojkrokové riešenie inverznej úlohy s ohľadom na možné klinické využitie.

Výsledky práce ukazujú, že poloha významných elektród závisí od polohy srdcového zdroja, pričom neboli zaznamenané výrazné rozdiely v polohe významných elektród medzi homogénnymi a nehomogénnymi modelmi hrudníka. Použitie 32 a 64 najvýznamnejších elektród poskytuje porovnateľnú presnosť inverzného riešenia (LE 26,4 $\pm$ 9,9 mm a 27,3 $\pm$ 5,6 mm) ako pri použití všetkých elektród (LE 24,7 $\pm$ 3,7 mm). Ďalej sme analyzovali optimálne umiestnenie elektród, ktoré ukázalo, že strategicky umiestnených 64 elektród môže poskytnúť presnejšie riešenie (LE 27,0 $\pm$ 14,3 mm) ako použitie všetkých elektród (LE 28,8 $\pm$ 11,9 mm). Dvojkrokové inverzné riešenie má schopnosť poskytovať presné výsledky s použitím 32 elektród (LE 28,8 $\pm$ 14,5 mm), s potenciálom posúdenia spoľahlivosti prvého inverzného riešenia.

Tieto výsledky poukazujú na možnosť použitia zníženého počtu významných elektród na riešenie inverzného problému, čo by umožnilo implementovať pre pacienta pohodlnejšiu konfiguráciu elektród.

Contents

Motivation

The incidence of cardiovascular diseases is increasing, with more than 670,000 people under cardiological observation in Slovakia in 2021 [1]. This dissertation thesis focuses on a specific cardiac condition known as premature ventricular contraction (PVC), characterized by premature electrical impulses causing extra heartbeats. A subset of patients with PVCs undergo a procedure called radiofrequency catheter ablation (RFA), which aims to identify the sources of these premature impulses and eliminate them through the application of radiofrequency energy [2].

The advent of computer technology has enabled us to investigate heart function without invasive intervention, which has been of great interest in cardiology. The solution of the inverse problem of electrocardiography non-invasively identifies the PVC source, reducing RFA procedure duration and radiation exposure [3]. Multiple steps need to be undertaken to solve the inverse problem, starting from the body surface potential mapping (BSPM) and geometrical modelling, leading up to the application of advanced mathematical and computational methods [4]. Currently, there is no universally accepted standard for BSPM and research groups use multi-lead ECG systems with varying numbers of electrodes (e.g., 64 [5], 128 [6], 192 [7]) placed at different positions on the torso's surface. There are other systems with varying electrode numbers, some having fewer and others having more than the ones mentioned here.

Attaching numerous electrodes to a patient's torso is a time-consuming and complex procedure. This complexity is one of the reasons why the practical application of the inverse problem in clinical settings has been limited. Furthermore, skin issues, the presence of other electrodes (e.g., defibrillation or cardiac mapping system electrodes), and the need to exclude malfunctioning electrodes further complicate the process [3], [8]. As a result, a question arises regarding the ideal number of electrodes and their ideal placement on the torso.

Several studies investigated the quantity and spatial arrangements of electrodes required to address the inverse problem accurately [9], [10], [11], [12], [13], [14]. All of the studies showed that the inverse problem can be solved accurately using a smaller number of electrodes. However, the absence of a universally accepted "gold standard" is notable, given the considerable variability in multi-lead ECG systems and inverse problem-solving methods employed by different research teams. Therefore, the aim of this thesis is to explore the positions of the most significant electrodes for a solution of the inverse problem assuming a single dipole cardiac source, method used in our laboratory. Furthermore, we would like to determine if the inverse problem can be solved accurately using a reduced number of significant electrodes. Thus, the four objectives of this dissertation thesis are:

- **the proposal of a method for the estimation of the significance of the individual torso electrodes** of the multi-lead ECG measuring system for the solution of the forward and the inverse problem of electrocardiography (premature ventricular contraction localization) using a single dipole;

- **the investigation of the positions of the significant torso electrodes** of the multi-lead ECG measuring system for the solution of the inverse problem of the electrocardiography considering different positions of the cardiac source and finite homogeneous and inhomogeneous torso model as a volume conductor;

- **the study of the influence of using the significant torso electrodes** of the multi-lead ECG measuring system **on the accuracy of the inverse solution** of the electrocardiography for the known position of the cardiac source;

- **the methodology proposal** for the solution of the inverse problem of electrocardiography based on incorporating the most significant torso electrodes **for clinical data**.

To accomplish the goals of this dissertation thesis, the steps shown in Figure 1 are undertaken. The methodology used to accomplish the objectives of this dissertation thesis is briefly explained in the subsequent chapters. The order of the methodology descriptions is aligned with the individual steps stated here.

Figure 1: Individual steps of the dissertation thesis.

1 Data

The inverse problem of electrocardiography identifies the origin of the heart's electrical activity, such as PVC. Ensuring the accuracy of the inverse solution is crucial for its clinical applicability. This validation process involves comparing the solution with the ground truth data. In this work, we used two datasets with the known ground truth: Bratislava (13 cases) and EP Solutions (8 cases).

The Bratislava dataset was obtained by the Institute of Measurement Science, Slovak Academy of Sciences (IMS SAS) in collaboration with the National Institute for Cardiovascular Diseases, Slovakia. The EP Solutions dataset was obtained from the publicly available database EDGAR (https://www.ecg-imaging.org/edgar-database). Both datasets contain 3D geometrical models of the torso and heart, while the Bratislava dataset contains also models of lungs and blood cavities. The BSPs were recorded in 128 torso electrodes for the Bratislava dataset using measuring system ProCardio 8 [6] and in 196 $\pm$ 28 electrodes for the EP Solutions dataset. In the Bratislava dataset, the position of the ground truth corresponds to the ablation area from the RFA procedure, whereas in the EP Solutions dataset, it corresponds to the tip of the pacemaker's electrode in the right ventricle (RV) and the left ventricle (LV). Additional information about the Bratislava dataset can be found in Svehlikova et.al [15], and about the EP Solutions dataset in Potyagaylo et al. [16].

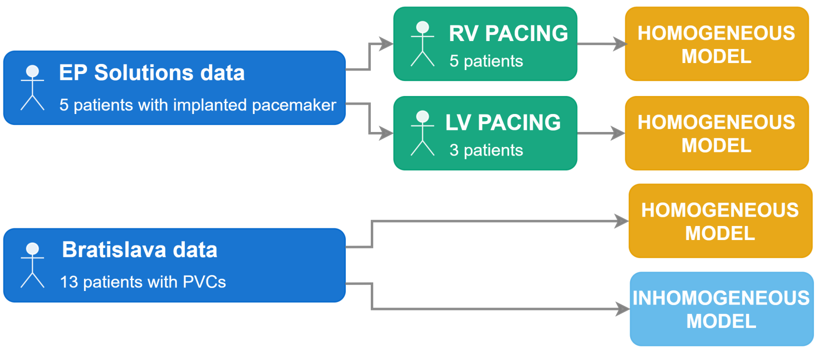

Figure 2: Datasets used in this study.

2 Inverse Solution

The inverse problem of electrocardiography is a non-invasive method that reconstructs the heart's electrical activity using BSPs along with the patient-specific models of the heart, torso, and internal inhomogeneities created using CT or MRI scans [17].

Let S_C be a vector of unknown cardiac electric sources, Φ_B be a vector of electric potentials measured on the body surface and T^+ be a Moore-Penrose pseudoinverse of the transfer matrix T. Then, the inverse problem is described by the linear equation [17], [18] as

$$S_C = T^+ \Phi_B. \tag{1}$$

A single dipole cardiac source is used in this dissertation thesis for the solution of the inverse problem. Each cardiac source S_C (represented by a single dipole) has a fixed position within the heart. The positions of dipoles correspond to the nodes of the 3D triangle mesh of the heart's endo-epicardium. The position of the cardiac source S_C that describes the input BSPs as well as possible is determined by choosing the one that results in the lowest relative residual error (RRE) during the time interval from the onset of depolarization up to 30 ms. The hypothesis is that during this short time, the active region of the heart is sufficiently small to be modelled as a single dipole. The RRE is calculated for every possible position of cardiac source S_C in each time step $t \in \langle 1, 30 \rangle$ ms as the normalized difference between the measured map Φ_B and the map computed by that dipole. The inverse solutions are computed using homogeneous and inhomogeneous volume conductors that incorporate lungs filled with air and heart cavities filled with blood. More details about the inverse solution using a single dipole cardiac source can be found in [15].

The accuracy of the inverse solution is determined by the localization error (LE), which measures the Euclidean distance between the ground truth and the inverse solution.

3 A Greedy Selection

3.1 Methods

Our inverse solution searches for the best position of the cardiac source (single dipole) based on a given set of input data. The significance of each torso electrode was determined by analyzing the singular values obtained from the singular value decomposition (SVD) of the transfer matrix. The transfer matrix contains information about the geometrical and electrical properties of the torso volume conductor and can be considered as a linear operator transferring the data from the heart to the torso. The transfer matrix is computed using the boundary element method (BEM). Supposing m electrodes located on the torso surface and n single dipoles described by the 3 orthogonal components of the dipole moment, the transfer matrix has size $T \in \mathbb{R}^{m \times 3n}$. The significance of the torso electrodes is estimated for a selected position of a single dipole cardiac source. The size of the transfer matrix computed for a given position of the cardiac source is $T \in \mathbb{R}^{m \times 3}$ and the SVD can be written as

$$T = U \Sigma V^T, \tag{2}$$

where $U \in \mathbb{R}^{m \times m}$, $V \in \mathbb{R}^{3 \times 3}$ and $\Sigma \in \mathbb{R}^{m \times 3}$. The rank r of matrix T is 3. Using only r columns of U and V assigned to nonzero singular values, the compact SVD of the transfer matrix T can be written as

$$T = U_r \Sigma_r V_r^T, \tag{3}$$

where $U \in \mathbb{R}^{m \times 3}$, $V \in \mathbb{R}^{3 \times 3}$ and $\Sigma \in \mathbb{R}^{3 \times 3}$. The matrix $\Sigma \in \mathbb{R}^{3 \times 3}$ has 3 singular values on its diagonal sorted such as $\sigma_1 \geq \sigma_2 \geq \sigma_3$. The singular values provide insight into the significance of the electrode space (columns of U) and cardiac (dipole) source space (columns of V) in the system. With respect to the torso, the singular values tell us how the given dipole with 3 orthogonal components is observable by the electrodes on the torso. This is measured by the magnitude of the singular values – the higher the values, the higher the observability of the cardiac source. Three criteria were tested for the estimation of the significance of the torso electrodes, all derived from the singular values.

Criterion A minimizes the conditioning number as

$$A = min(\frac{\sigma_1}{\sigma_3}), \tag{4}$$

where σ_1 is the largest singular values and σ_3 is the smallest singular value. A large condition number indicates that the matrix is ill-conditioned and difficult to invert, while a small condition number indicates that the matrix is well-conditioned and more stable and accurate in computational algorithms [19].

Criterion B maximizes the product of the singular values as

$$B = max(\sigma_1 * \sigma_2 * \sigma_3), \tag{5}$$

and represents the (scaled) volume of the image of the unit ball under the linear operator T. This criterion is sensitive to the small singular values since one small singular value will affect the result of the multiplication.

Criterion C aims to maximize the total variance, which is represented by the sum of the singular values as

$$C = max(\sigma_1 + \sigma_2 + \sigma_3). \tag{6}$$

Criterion C is a generalization of the matrix's rank, which is a measure of its dimensionality. Unlike criterion B, criterion C exhibits reduced sensitivity to minor singular values, as it involves the calculation of the sum of these values [20].

Testing all of the possible combinations of electrodes that would meet the given criteria the best is computationally infeasible. Hence, a greedy selection of torso electrodes is implemented [21]. Two tasks are executed for each criterion as described below and shown in Figure 3. First, the best combination of 4 electrodes is selected from all possible combinations (e.g., 10,668,000 combinations of 4 from 128 torso electrodes). This involves computing the SVD of 10,668,000 transfer matrices of size $T \in \mathbb{R}^{4\times3}$ to find the starting combination of 4 electrodes. Next, a greedy selection is implemented where one electrode at each step is added to the previously selected set of electrodes to satisfy the criterion as best as possible. The electrodes are chosen without repetition, and this process continues until all available electrodes are used [21].

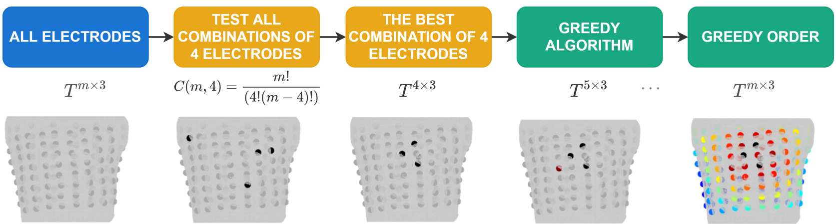

Figure 3: Pipeline for the implementation of the greedy algorithm, where m represents the number of torso electrodes. The electrodes within the starting combination of 4 electrodes might have a posterior localization and this pipeline's visualization is limited to the anterior view.

3.2 Results

The selection of the torso electrodes begins by selecting the best combination of 4 electrodes from all possible combinations. The electrodes can be repeated within these combinations; thus, some may occur more frequently within the top combinations than others. The occurrence of electrodes within the top 1% of combinations computed for patient P001 from the Bratislava dataset and homogeneous torso model is shown in Figure 4. Assuming 128 electrodes, there are a total of 10,668,000 possible combinations of 4 electrodes, and 1% from all combinations is 106,680. An observable pattern is seen wherein certain electrodes appear more frequently within the top 1% of combinations. The electrodes with higher occurrence are situated anteriorly in the upper half of the torso, particularly in the vicinity of the sternum.

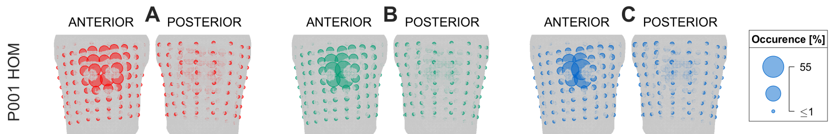

Figure 4: The occurrence of electrodes within the top 1% of combinations from all possible combinations of 4 for patient P001 from the Bratislava dataset and homogeneous (HOM) torso model and three considered criteria. The legend on the right corresponds to all three criteria.

Figure 5 illustrates the greedy order of electrodes for patient 024 from the EP Solutions dataset. The significance of the electrodes was computed for the ground truth corresponding to the position of the pacemaker's electrode in the RV. The initial combination of 4 electrodes is represented in black, while the most significant electrodes are highlighted in red, and the least significant ones are indicated in blue. The greedy order is depicted for all 3 criteria described above. Distinct greedy orders are computed, primarily for criterion A, whereas similarities are observed between criteria B and C. The most significant electrodes for criteria B and C are predominantly localized on the anterior side of the torso, while for criterion A, they are found on the posterior side of the torso. The dissimilarity between criterion A and criteria B and C can be attributed to the exclusion of the second singular value in criterion A, potentially resulting in the loss of a significant information transfer component from the dipole space to the electrode space.

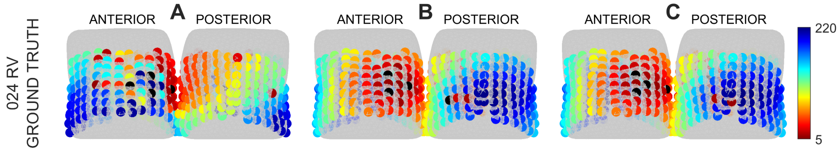

Figure 5: Greedy order of electrodes for patient 024 from the EP Solutions dataset computed for the position of the pacemaker's electrode in the RV.

The transfer matrix contains information on how the electrical sources inside the heart contribute to the BSPs on the torso surface. Each row of the transfer matrix represents the sensitivity of the torso electrode to the electrical activity from different locations of the heart. Thus, the significance of the torso electrodes obtained by the proposed methodology can be compared to the properties of the measured BSPs. Figures 5 and 6 show that there is a relationship between the positions of the most significant electrodes computed for criteria B and C and the electrodes with the highest signal power. However, even when the positions of the most significant and most powerful electrodes overlap, it becomes apparent that only a few electrodes show notably higher power. As a result, determining the precise electrode order based solely on signal power may present challenges.

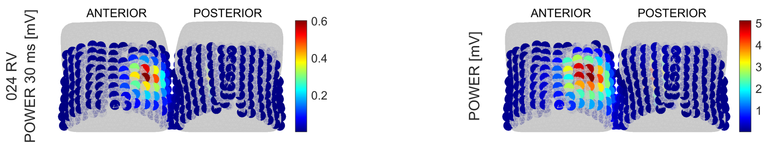

Figure 6: The signal power of the electrodes for patient 024 from the EP Solutions dataset computed for the signals obtained during RV pacing. The signal power is depicted for the earliest phase of the cardiac cycle (left) and the whole cycle (right).

Computational Complexity

The algorithm for the estimation of the significance of the torso electrodes was implemented in MATLAB (MATLAB R2021b and MATLAB R2022b) and tested using a laptop computer equipped with processor Intel Core i7-1065G7 CPU @ 1.30 GHz, 1498 Mhz, 4 cores and graphical card NVIDIA GeForce MX350. First, the best combination of 4 electrodes is selected from all possible combinations. Assuming 128 electrodes, the computational time for determining the best set of 4 electrodes is approximately 6.4 seconds. Then, the greedy order of the electrodes is determined by the greedy algorithm in approximately 1.6 seconds.

4 Unraveling the Positions of Significant Electrodes

4.1 Methods

The second objective of this dissertation thesis is to investigate the significance of individual torso electrodes of a multi-lead ECG measuring system. Therefore, the significance of the torso electrodes was investigated using the proposed approach described in Chapter 3 for homogeneous and inhomogeneous torso models and different origins of cardiac activity. In this part of the work, the significance of the torso electrodes for distant as well as neighbouring cardiac sources was also investigated.

4.2 Results

Our findings suggest that the positions of the most significant electrodes, calculated for the same cardiac source but using homogeneous and inhomogeneous torso models, are not identical but exhibit high similarity, as shown in Figure 7 for criteria B and C. When comparing homogeneous and inhomogeneous torso models, 10 ± 7 (A), 26 ± 2 (B), and 24 ± 2 (C) electrodes were identified as identical among the 32 most significant ones. Regarding the 64 most significant electrodes, it was 35 ± 10 (A), 56 ± 3 (B), and 51 ± 3 (C) electrodes. Consequently, using a homogeneous torso model alone would not result in a substantial variance in the locations of the most significant electrodes for criteria B and C.

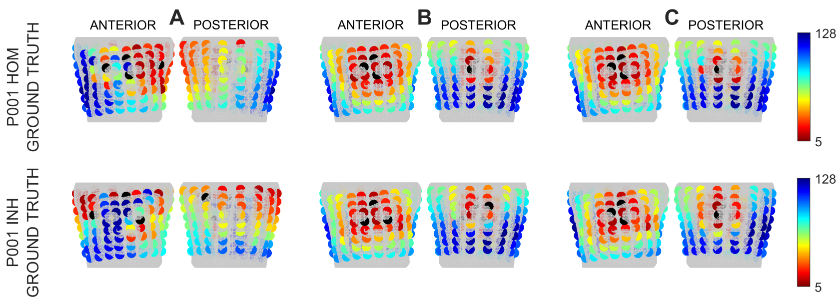

Figure 7: Greedy order of electrodes for patient P001 from the Bratislava dataset computed for the position of the ground truth assuming homogeneous (HOM) (upper panel) and inhomogeneous (INH) (lower panel) torso models.

The study underscores the dependency of the significant electrodes' locations on the position of the cardiac source as shown in Figure 8. When the cardiac source is situated at the apex of the RV, the significant electrodes tend to be located predominantly anteriorly. In contrast, if the cardiac source is located in the LV, closer to the posterior torso, significant electrodes are found in both the anterior and posterior regions of the torso. Nonetheless, in both cases, the significant electrodes are primarily situated on the left side of the torso, closer to the heart.

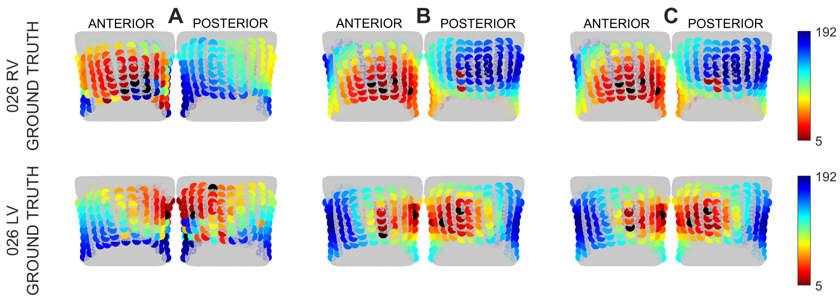

Figure 8: Greedy order of electrodes for patient 026 from the EP Solutions dataset computed for the position of the pacemaker's electrode in the RV (upper panel) and LV (lower panel).

Moreover, the study revealed that the most significant electrodes are identified in similar areas of the torso when assuming neighbouring cardiac sources, as shown in Figure 9 for criteria B and C. The significance of the torso electrodes was computed for 161 cardiac sources within the sphere with a diameter of 30 mm. Then, the occurrence of each electrode among the top 15% of the most significant ones for all cardiac sources within the sphere was computed. When considering criterion A, approximately 90% of all electrodes were found within the top 15% of the most significant electrodes. However, for criteria B and C, a smaller proportion of electrodes, specifically 45% and 41% of all electrodes, occurred within the top 15% of the most significant ones [22]. The study's findings suggest that criteria B and C are more effective in identifying the significance of electrodes for our specific task compared to criterion A. This is due to the fact that under criteria B and C, the most significant electrodes remain localized in specific areas rather than being scattered throughout the entire chest.

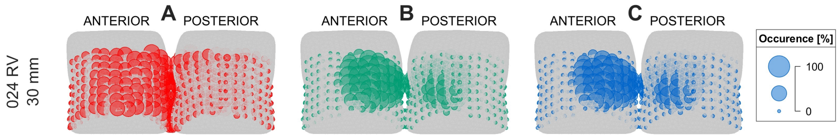

Figure 9: Occurrence of 15% of the most significant electrodes for patient 024 from the EP Solutions dataset computed for all cardiac sources within 30 mm sphere for RV pacing. The legend on the right corresponds to all 3 criteria.

5 Significant Electrodes and Inverse Solution Accuracy

5.1 Methods

One of the objectives of this thesis was to investigate how the significant electrodes affect the accuracy of the inverse solution. Therefore, we solved the inverse problem according to the Equation 1 using 4 up to the complete set of electrodes sorted according to its significance (1^{st} scenario), and then in reverse order (2^{nd} scenario) for the EP Solutions dataset [23].

The LE was computed for each inverse solution, using 4 up to the full number of electrodes. For the EP Solutions dataset, the number of torso electrodes varied among patients from 164 to 229. In addition, the LE obtained when 32, 64 and 128 electrodes were used was compared with the LE obtained when all electrodes were used for the inverse solution as used in [12], [24].

5.2 Results

The LE values obtained using 4 up to the complete set of electrodes are depicted in Figure 10. Using a small number of significant electrodes, similar accuracy than using all electrodes can be obtained (upper panel). Using a small number of insignificant electrodes, a high deterioration in localization accuracy is observed (lower panel).

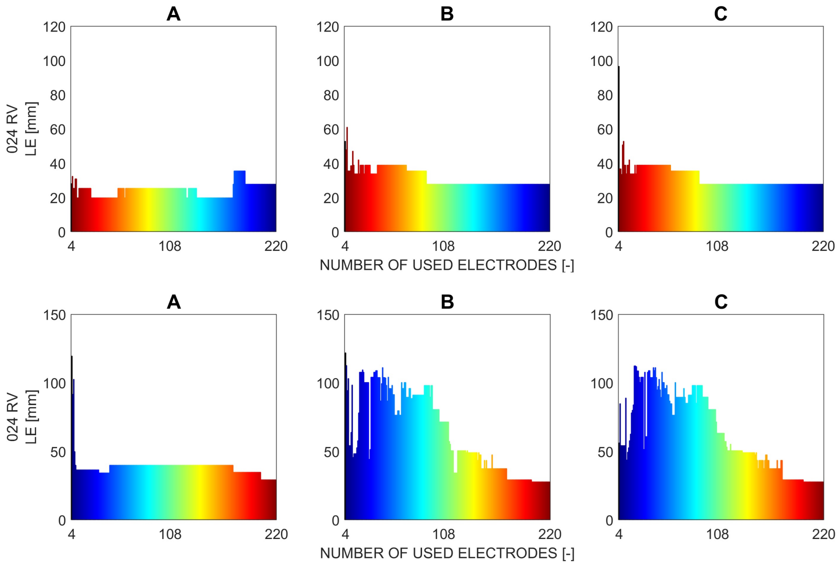

Figure 10: The LE for patient 024 from the EP Solutions dataset for the 1^{st} (upper panel) and 2^{nd} (lower panel) scenario computed for the position of the ground truth in the RV.

Using a subset of significant electrodes (1^{st} scenario) can result in comparable accuracy to using all the torso electrodes, and in certain cases, even an improvement in localization was observed as shown in Figure 11. When using 32 electrodes, the most accurate localization of 26.4 ± 9.9 mm was observed for criterion C. With 64 electrodes, the best localization of 27.2 ± 6.2 mm was achieved for criterion B, and using 128 electrodes, it was 24.3 ± 2.9 mm for criterion A. The LE obtained using all electrodes was 24.7 ± 3.7 mm. Thus, these findings suggest that using a smaller number of significant electrodes does not noticeably deteriorate the accuracy of the inverse solution, and similar results can be obtained as using the complete set of electrodes.

However, it was also evident that using insignificant electrodes (2^{nd} scenario) led to a substantial deterioration in localization accuracy. For the 2^{nd} scenario when using 32 electrodes, the most precise localization of 35.7 ± 13.0 mm was achieved for criterion A. Interestingly, criterion B exhibited substantial degradation, resulting in an LE of 78.2 ± 30.7 mm. With 64 electrodes, criterion A continued to yield the most favourable outcomes, with an LE of 33.1 ± 16.2 mm, while criterion C exhibited an LE of 75.9 ± 36.2 mm. Moving on to 128 electrodes, the LE of 25.1 ± 16.4 mm was recorded for criterion B. Although there were cases where the LE improved for the 2^{nd} scenario, the results indicate that using a smaller number of insignificant electrodes worsens the outcome of the inverse problem solution. Thus, if we would like to reduce the number of electrodes used for the solution of the inverse problem it becomes essential to thoroughly examine the optimal placement of the reduced set of the electrodes.

This study confirmed that the inverse problem can be solved using a smaller number of electrodes. However, it is crucial to prioritize the use of the most informative electrodes, as demonstrated in the 1^{st} scenario.

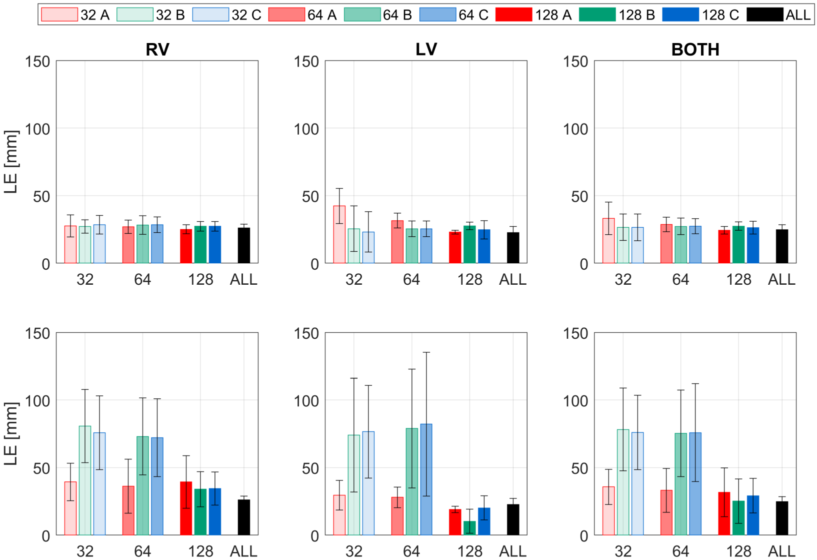

Figure 11: The mean LE and standard deviation in mm obtained using 32, 64, 128 and the full set of 196 ± 28 electrodes (ALL) for patients from the EP Solutions dataset and the 1^{st} (upper panel) and 2^{nd} scenario (lower panel). The mean and standard deviation were calculated for RV and LV pacing, as well as for both combined.

6 A Two-Step Inverse Solution

6.1 Methods

Estimation of the significance of the torso electrodes proposed in this dissertation thesis is based on the SVD of the transfer matrix computed for a given position of the cardiac source, i.e, ground truth. However, the ground truth is not known in clinical practice, making this approach impractical. To overcome this limitation while integrating the method for assessing the significance of torso electrodes, a two-step inverse solution is introduced in Ondrusova et al. [20]. The implementation of the two-step inverse solution is described below and shown in Figure 12:

- First, the inverse solution is solved using information from all torso electrodes, thus using the BSP map $\Phi_B \in \mathbb{R}^{m\times 30}$ and the transfer matrix $T \in \mathbb{R}^{m\times 3n}$ where m is the number of torso electrodes and n is the number of cardiac sources.

- Then, the significance of the electrodes for the first inverse solution is determined from singular values of the transfer matrix $T \in \mathbb{R}^{m\times 3}$ corresponding to the inversely computed origin of the cardiac activity. The significance of the electrodes is derived from all three criteria.

- Second, the inverse solution is computed using from 4 up to the full set of electrodes m sorted based on its greedy order (sorted from the most to the least significant), thus using 4 up to all rows of BSP map $\Phi_B \in \mathbb{R}^{(4:\,1:\,m)\times 30}$ and the transfer matrix $T \in \mathbb{R}^{(4:\,1:\,m)\times 3n}$.

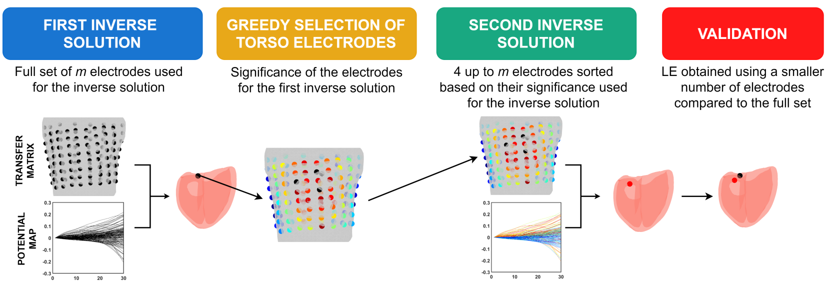

Figure 12: The pipeline of the two-step inverse solution.

The LE in each step was computed in order to validate the two-step inverse solution. Furthermore, the LE obtained using 32 and 64 most significant electrodes was compared to the LE obtained using all 128 torso electrodes as used in the Bratislava dataset.

6.2 Results

Using a two-step inverse solution, the improved localization can be obtained using a smaller number of significant electrodes compared to using a complete set of electrodes, as can be seen for patient P008 in Figure 13 for criteria B and C. Nevertheless, in certain cases, deterioration can be observed, as seen for patient P010.

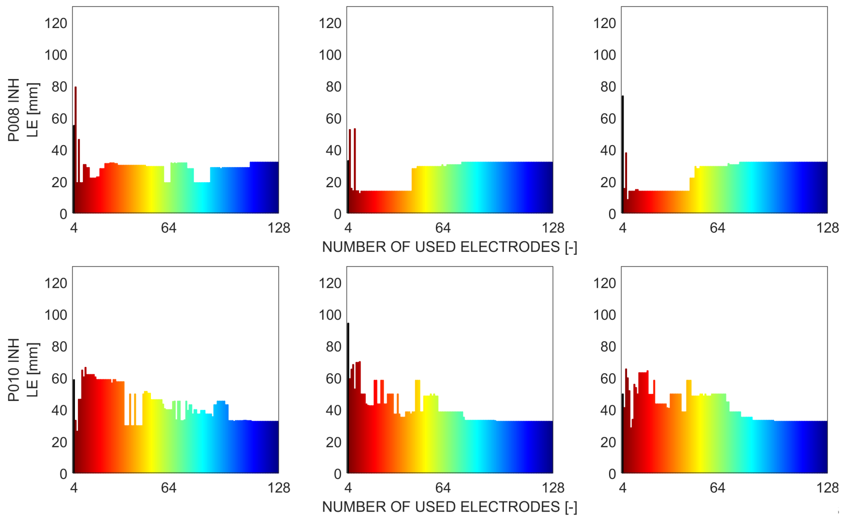

Figure 13: The LE for patients P008 and P010 from the Bratislava dataset for the 2^{nd} step of the two-step inverse solution for inhomogeneous (INH) torso model.

It appears that using a small number of significant electrodes (e.g., fewer than 16) could potentially be enough for an accurate solution of the inverse problem as demonstrated for patient P008. The analysis of RRE values on the heart's mesh in Figure 14 aimed to explore the stability and localization of the criterion for minimal RRE. Each node on the mesh corresponds to a specific dipole position, and an RRE value is assigned to it. The node with the lowest RRE is considered the winning node, as it best describes the electrical activity. When using 4-16 electrodes, identifying the node with the lowest RRE becomes challenging due to the similarity of RRE values for all possible dipole positions on the heart. However, it is easier to select the node with the lowest RRE from a localized area of low RRE values with the increasing number of used electrodes.

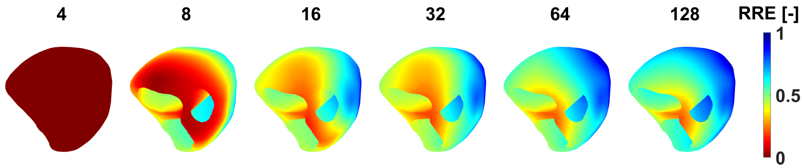

Figure 14: The distribution of RRE values on the endo-epicardial ventricular triangulated surface when a specific number of electrodes were used for the inverse solution.

The mean LE values and their corresponding standard deviations for each criterion and using 32, 64 and a complete set of 128 electrodes were computed for all patients as shown in Figure 15. Considering both torso models, the lowest LE of 28.8 $\pm$ 14.5 mm was observed for criterion C using 32 electrodes. Using 64 electrodes, the lowest LE of 29.4 $\pm$ 12.0 mm was observed for criterion B. The worst performance of 32.6 $\pm$ 19.9 mm was observed under criterion A when using 32 electrodes. The LE obtained using the full set of electrodes was 28.8 $\pm$ 11.9 mm.

The outcomes derived from the two-step inverse solution indicate that we can indeed achieve accurate results in solving the inverse problem with a reduced number of torso electrodes. However, this approach necessitates the use of all electrodes in the first step and for some patient cases, a decline in localization accuracy was observed. Nevertheless, this approach could serve as a tool to assess the reliability of the inverse solution. If the result obtained from the inverse solution using a smaller set of significant electrodes considerably deviates from that using all electrodes, it may raise concerns about the accuracy of the first solution.

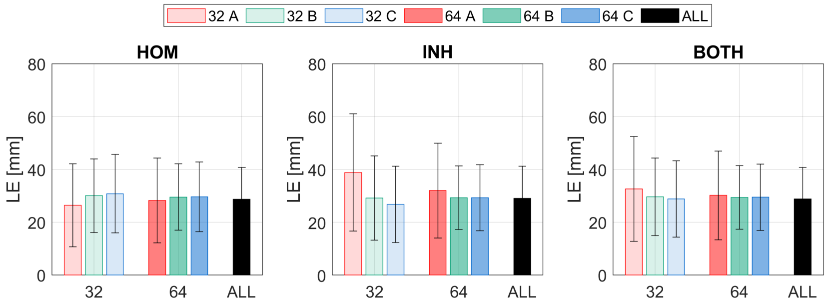

Figure 15: The mean LE and standard deviation in mm obtained using 32, 64 and full set of 128 electrodes (ALL) for patients from the Bratislava dataset for the 2^{nd} step of the two-step inverse solution using both torso models. The mean and standard deviation were computed separately for the homogeneous (HOM) and inhomogeneous (INH) models, as well as for them both (BOTH).

7 Optimal Lead Placement

7.1 Methods

All analyses described above investigate the significance of the torso electrodes for a given position of the cardiac source. Nevertheless, one would like to determine the optimal lead placement irrespective of the cardiac source's position. This study builds upon the goals of this dissertation thesis.

To address this, we calculated the significance of the torso electrodes for all nodes (cardiac sources) within the ventricular mesh, as depicted in Figure 16. The study was carried out on two patients from the Bratislava dataset, namely, P001 and P004, due to their distinct geometrical properties as shown in Figure 17. To speed up the computations, the high-density meshes for these patients were transformed into lower-density, regularly spaced meshes, as shown in Figure 16. The ventricular mesh for patient P001 consisted of 550 nodes, while the mesh for patient P004 had 495 nodes. The significance of the electrodes was computed for cardiac sources in all these nodes. Thus, 1045 greedy orders (550 + 495 cardiac sources) were computed. For each cardiac source, the most significant electrodes were identified and the optimal placement of 32 and 64 electrodes was derived.

Lastly, the 32 and 64 optimally placed electrodes were used for the inverse solutions for all 13 patients from the Bratislava dataset, and the results were compared to the inverse solutions using all torso electrodes.

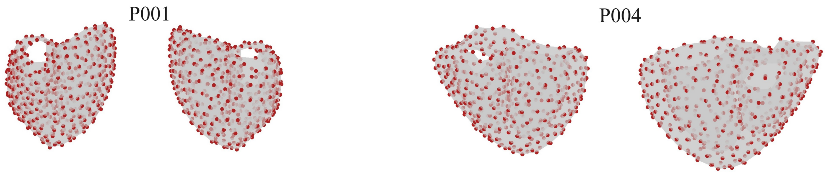

Figure 16: The positions of all cardiac sources on ventricular mesh (red dots) for the investigation of the significance of the torso electrodes for the homogeneous torso model and patient P001 (left) and P004 (right) from the Bratislava dataset. For each patient, the anterior and posterior view is depicted.

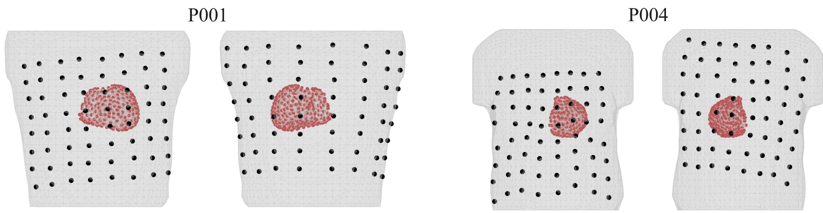

Figure 17: The positions of the cardiac sources (red dots) for the investigation of the significance of the torso electrodes for the homogeneous torso models for patients P001 (left) and P004 (right) from the Bratislava dataset. Torso electrodes are depicted in black. For each patient, the anterior and posterior view is depicted.

7.2 Results

The optimal 32-lead and 64-lead placements were determined by identifying the electrodes that appeared most frequently among the most significant ones for both patients and all cardiac sources. These lead sets are depicted in Figure 18. For criteria B and C, the analysis reveals that the 32 most significant electrodes are consistently located in the anterior left half of the torso, irrespective of the potential positions of the cardiac source. The 64 most significant electrodes are predominantly situated in the left anterior region, with the exception of some electrodes on the right lateral torso. A few electrodes from this set are also identified posteriorly within the left half of the torso.

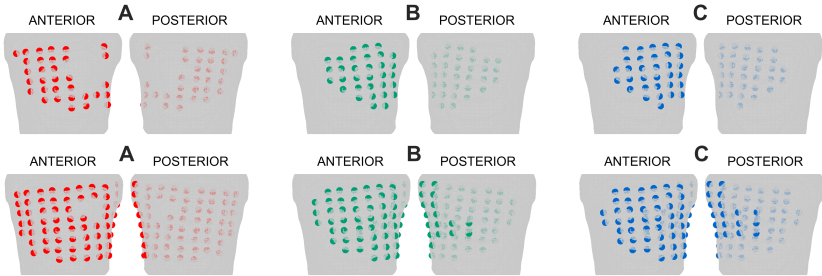

Figure 18: The optimal placement of 32 electrodes (upper panel) and 64 electrodes (lower panel) computed for all possible positions of the cardiac source.

The inverse solution was computed for each patient from the Bratislava dataset using 32 and 64 electrodes as depicted in Figure 18. The positions of the used electrodes were consistent across all patients. The results for each criterion and both torso models are summarized in Figure 19. When using 32 electrodes, a higher LE was evident for each torso model and each criterion. Using 64 electrodes, a deteriorated LE was observed compared to the use of all electrodes for criterion A. Interestingly, criteria B and C displayed improved localization. The most favourable outcomes were achieved with criterion C, where the LE improved to 27.4 $\pm$ 14.6 mm, as opposed to 28.6 $\pm$ 12.1 mm when using all electrodes and a homogeneous torso model. Similarly, the LE improved for the inhomogeneous model, reducing to 26.6 $\pm$ 14.6 mm compared to the 29.0 $\pm$ 12.1 mm obtained using the full set of electrodes.

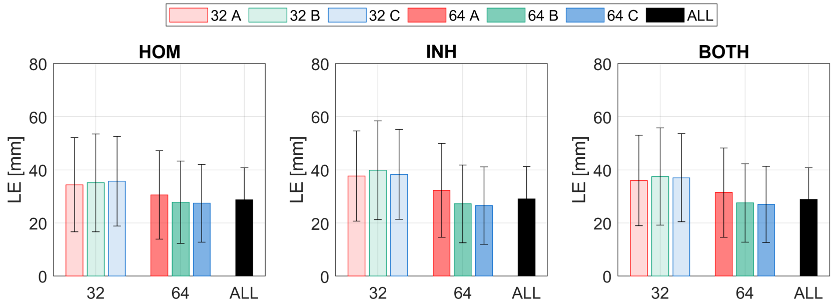

Figure 19: The mean LE and standard deviation in mm obtained sing 32 and 64 optimal electrodes and the full set of 128 electrodes (ALL) for patients from the Bratislava dataset using both torso models. The mean LE values and standard deviations were computed for the homogeneous (HOM) and inhomogeneous (INH) models separately, as well as for them both (BOTH).

Two-Step Inverse Solution vs Optimal Lead Placement

In this dissertation thesis, we solved the inverse problem by using two different approaches. First, we solved the inverse solution using the most significant electrodes determined based on the approximate position of the cardiac source, therefore using a two-step inverse solution as proposed in Chapter 6. After summarizing all the results presented here, it is apparent that criterion A produced the least favourable outcomes. Therefore, the following description focuses on criterion C which showed similar, but slightly better results than criterion B.

The two-step inverse solution outperformed the optimal electrode placement when limited to 32 electrodes as depicted in Figure 20. With the two-step inverse solution, the mean LE of 28.8 ± 14.9 mm was comparable to the LE 28.8 ± 11.9 mm obtained using the full electrode set. In cases such as the inhomogeneous model and criterion C, an improvement in LE to 26.8 ± 14.4 mm was observed compared to the LE 29.0 ± 12.1 mm obtained using the full set of electrodes.

Interestingly, when using 64 electrodes, the optimal electrode placement exhibited superior performance with the LE 27.0 ± 14.3 mm compared to the two-step inverse solution with the LE 29.5 ± 12.6 mm, and it also outperformed the solution using all electrodes with the LE 28.8 ± 11.9 mm. Nevertheless, the differences in mean LE between the two-step inverse solution and optimal electrode placement were relatively modest.

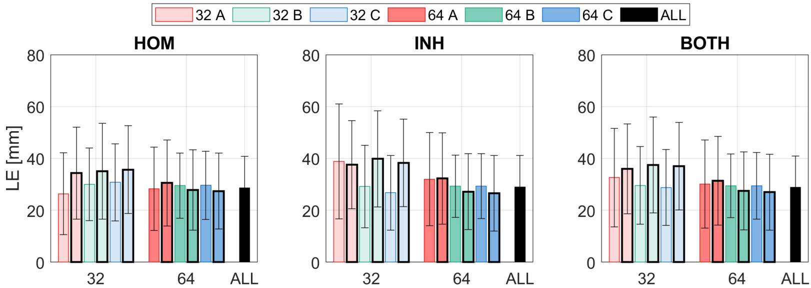

Figure 20: The mean LE and standard deviation in mm obtained using 32 and 64 optimal electrodes (bars in black boxes) and 32 and 64 electrodes of the two-step inverse solution and the full set of electrodes (ALL) for patients from the Bratislava dataset. The mean LE values and standard deviations were computed for the homogeneous (HOM) and inhomogeneous (INH) models separately, as well as for them both (BOTH).

Unfortunately, in this study, the optimal placement was determined for all potential positions of the cardiac source for just two patients. Nevertheless, for future research, it would be intriguing to ascertain the optimal placement using a larger dataset of patients to encompass a broader range of geometrical variations among patients.

Despite the limitations, the results of the two-step inverse solution and the optimal placement studies suggest that the inverse solution can be solved accurately using a smaller number of electrodes.

Conclusion

There is a need to broaden our knowledge about methods for the non-invasive assessment of the state of the patient's heart due to the rising number of patients with cardiovascular diseases. In this dissertation thesis, we focused on solving the inverse problem of electrocardiography that can help us determine the origin of ectopic cardiac activity, such as PVCs. Despite its potential, the solution of the inverse problem still needs to be integrated into routine clinical procedures. One drawback to its adoption is the requirement to use a substantial number of torso electrodes to capture the heart's electrical activity. Reducing the number of electrodes can enhance the practicality of this approach in clinical settings.

Therefore, in this thesis, we explored the positions of the most significant electrodes for the solution of the inverse problem. Additionally, we investigated whether we could achieve an accurate inverse solution using a selected subset of significant electrodes.

First, we proposed a methodology to estimate the significance of torso electrodes based on an analysis of the transfer matrix computed for a specific cardiac source position. Three criteria were suggested using singular values from the SVD of the transfer matrix. Electrode selection uses a greedy algorithm to minimize computational demands. The criterion that computes the sum of singular values demonstrates the best potential to yield optimal results. This is because it encompasses all singular values and is less susceptible to smaller values' influence than the multiplication-based criterion. Therefore, the results summarized in this chapter concentrate on the criterion that maximizes the sum of singular values.

Second, we applied the proposed approach to examine the locations of the most significant electrodes within both homogeneous and inhomogeneous volume conductors, considering various positions of the cardiac source as summarized in Chapter 4. Homogeneous and inhomogeneous torso models yield similar positions of significant electrodes, indicating just minor differences. Thus, using only a homogeneous model doesn't significantly change the positions of significant electrodes compared to an inhomogeneous model. Furthermore, the study underscores the dependency of the significant electrode positions on the position of the cardiac source, as shown for RV and LV ground truth. Moreover, the study revealed that the most significant electrodes are identified in similar areas of the torso when assuming neighbouring cardiac sources, thus building the foundations for the two-step inverse solution.

Subsequently, we analysed whether the inverse problem could be accurately solved using the significant torso electrodes. As elaborated in Chapter 5, using a subset of significant electrodes can result in comparable accuracy to using all the torso electrodes, and in certain cases, even an improvement in localization was observed. However, it was also evident that the use of insignificant electrodes led to a substantial deterioration in localization accuracy.

The significance of the torso electrodes was determined based on a specific cardiac source position, which is not known in clinical practice. To address this challenge, we introduced a two-step inverse solution as described in Chapter 6. However, this approach necessitates the use of all electrodes in the first step. Moreover, in some patient cases, a decline in localization accuracy was observed using a smaller number of electrodes. Nevertheless, this approach could serve as a tool to assess the reliability of the first inverse solution.

Using a two-step inverse solution approach, it becomes feasible to estimate the significance of the torso electrodes on an individual basis for each patient. Nevertheless, there is an interest in determining the "optimal lead placement". In order to identify this optimal placement of electrodes, the significance of the torso electrodes was calculated for all potential cardiac sources, evenly distributed on the endo-epicardial ventricular mesh, for two patients exhibiting distinct geometric properties as described in Chapter 7. The two-step inverse solution outperformed the optimal lead placement when using 32 electrodes. This outcome might be attributed to the absence of posterior electrodes in the 32-electrode configuration. However, the 64 optimal electrode setup exhibited superior performance over the two-step inverse solution. These findings are promising, indicating that using many electrodes over the whole torso may not be necessary. By strategically placing a subset of electrodes, it appears feasible to solve the inverse problem accurately. This would reduce patient preparation time, enhance comfort during measurements, and make the process quicker and more comfortable for a patient.

Despite various limitations, including geometric modelling constraints, manual transfer of ground truths, and a limited patient sample size, our study successfully showed that localizing PVC origin with the solution of the inverse problem, assuming a single dipole cardiac source, can be accurately achieved using a reduced set of significant electrodes. These findings underscore the potential for more efficient and patient-friendly electrode configurations in clinical practice.

Thesis Statement and Contributions

This dissertation thesis presents a novel method for evaluating the significance of torso electrodes in the context of the solution of the inverse problem of electrocardiography. Our approach is based on the analysis of the transfer matrix, which captures the torso's unique geometric and electrical properties. Specifically, we apply SVD to the transfer matrix associated with a specific position of the cardiac source.

While previous research has also used SVD of the transfer matrix to investigate the relation between cardiac sources and electrode configurations, to the best of our knowledge, our study represents a distinctive contribution. To elaborate, a prior study by Dössel et al. [10] applied SVD to transfer matrix computed using the finite element method in order to optimize the placement of torso electrodes. In contrast to our work, their analysis focused on the slope of the curve "σ_i over i", whereas we propose three distinct criteria for assessing the significance of torso electrodes. Furthermore, our research examined the significance of torso electrodes within measuring systems using varying numbers of torso electrodes. This analysis was conducted across different positions of the cardiac sources and involved multiple patients.

The method proposed in this dissertation thesis uses the greedy algorithm for assigning significance to the torso electrodes. This is in contrast to the studies in which they explored different electrode configurations by selecting different rows or columns of electrodes [11], [14]. A similarity can be observed between the incremental approach proposed by Gharbalchi No et al. [12] and the greedy algorithm introduced in this work. In both cases, electrodes are iteratively added to the previously selected set. However, it's important to emphasize that our method exclusively relies on the properties of the transfer matrix for electrode selection, while the sequential approach assesses the accuracy of the inverse solution itself. Furthermore, the order in which electrodes were selected was determined for patients with PVCs, which contrasts with the aforementioned study that used BSPs obtained by solving the forward problem using epicardial potentials recorded during ventricular pacing of a dog's heart. As evidenced, variations in ECGI accuracy exist between spontaneous PVCs and ventricular-paced beats [25]. Additionally, the mentioned study omitted patient-specific anatomical geometry, using only a homogeneous torso model. In contrast, the research conducted by Parreira et al. [14] involved multiple patients with PVCs and their specific anatomical geometries. However, this study did not include a method for selecting specific torso electrodes; instead, it focused on the selection of different electrode band configurations.

The primary emphasis of this study was on the identification of the most significant electrodes of the multi-lead ECG measuring system for the solution of the inverse problem. Currently, there is a focus on the solution of the inverse problem exclusively with electrodes of the standard 12-lead ECG [26], [27], [28]. However, our research by Ondrusova et al.[29] indicated that accurately solving the inverse problem with a single dipole cardiac source is not achievable using just 9 electrodes of the 12-lead ECG. Nonetheless, we demonstrated that better localization can be attained by using 9 significant electrodes as opposed to using 9 electrodes from the 12-lead ECG. The significance of the electrodes was determined using the approach proposed in this dissertation thesis.

The methodology proposed in this work for the determination of the significance of the torso electrodes, along with the analysis involving 8 patients with pacemakers and 13 patients with PVCs, including both homogeneous and inhomogeneous torso models, distinguishes this work from previously published research. Furthermore, we proposed a novel two-step inverse solution for use in clinical settings. Furthermore, a concept of optimal lead placement irrespective of the cardiac source's position was introduced. The results of this dissertation thesis underscore the potential of using a reduced number of significant electrodes to solve the inverse problem, making it feasible to implement more convenient and patient-friendly electrode configurations in clinical settings.

Resumé

Táto dizertačná práca je príspevkom k aktuálnej problematike rozvoja metód neinvazívneho hodnotenia stavu srdca pacienta. V tejto dizertačnej práci sme sa zamerali na riešenie inverzného problému elektrokardiografie, ktorý môže pomôcť určiť polohu zdroja ektopickej srdcovej aktivity, akou sú napríklad predčasné komorové extrasystoly (Premature Ventricular Contractions - PVC). Na riešenie inverzného problému elektrokardiografie je potrebné vytvoriť model hrudníka špecifický pre daného pacienta, zaznamenať elektrické potenciály z povrchu hrudníka pacienta a použiť vhodnú matematickú metódu. V porovnaní so štandardným meraním 12-zvodového EKG sa počas povrchového mapovania zachytí omnoho komplexnejší súbor údajov týkajúcich sa elektrickej aktivity srdca vďaka použitiu veľkého počtu elektród. Obvykle sa pri povrchovom mapovaní používajú desiatky až stovky hrudníkových elektród. V našom laboratóriu používame viaczvodový merací systém ProCardio 8 [6] so 128 elektródami.

Riešenie inverznej úlohy nemá zatiaľ rozsiahle klinické využitie v dôsledku nepraktickosti použitia desiatok až stoviek hrudníkových elektród. V tejto dizertačnej práci hľadáme umiestnenie najvýznamnejších elektród na hrudníku a analyzujeme potenciál využitia redukovanej sady významných elektród pre riešenie inverznej úlohy pomocou jedného dipólu (metódy používanej na našom pracovisku) na lokalizáciu zdroja nežiadúcej elektrickej aktivity v srdci.

Na dosiahnutie štyroch hlavných cieľov sme použili EP Solutions data (8 súborov dát zaznamenaných počas komorovej stimulácie) a Bratislava data (13 pacientov s spontánnymi PVC). Najskôr sme navrhli metodiku odhadu významnosti hrudníkových elektród. Tento prístup je založený na analýze prenosovej matice vypočítanej pre špecifickú polohu srdcového zdroja, teda výlučne s použitím geometrického a elektrického vzťahu medzi srdcovými zdrojmi a hrudníkovými elektródami. Navrhli sme tri kritériá na odhad významnosti pomocou singulárnych hodnôt získaných metódou singulárneho rozkladu (SVD) prenosovej matice: minimalizácia podielu najväčšej a najmenšej singulárnej hodnoty (číslo podmienenosti), maximalizácia súčinu, resp. súčtu singulárnych hodnôt. Výber elektród sme realizovali pomocou tzv. greedy algoritmu, čím sa minimalizovali výpočtové nároky. Kritérium čísla podmienenosti dáva iné a menej konzistentné výsledky ako zvyšné dve kritériá, pravdepodobne kvôli vynechaniu druhej singulárnej hodnoty, čo vedie k strate potenciálne dôležitej informácie. Kritérium, ktoré je založené na súčte singulárnych hodnôt má potenciál dosiahnuť optimálne výsledky, pretože zahŕňa všetky

singulárne hodnoty a je menej náchylné na vplyv menších hodnôt. Z tohto dôvodu sa numerické výsledky zhrnuté v tejto kapitole sústreďujú výlučne na kritérium, ktoré maximalizuje súčet singulárnych hodnôt, aj keď kritérium, ktoré maximalizuje súčin singulárnych hodnôt, tiež prinieslo porovnateľné výsledky.

Ďalej sme navrhnutú metódu použili na nájdenie najvýznamnejších elektród berúc do úvahy homogénne aj nehomogénne objemové vodiče a rôzne polohy srdcového zdroja. Výsledky ukazujú, že polohy najvýznamnejších elektród vykazujú vysoký stupeň podobnosti medzi homogénnym a nehomogénnym modelom hrudníka, avšak nie sú identické. Štúdia tiež poukazuje na závislosť polôh významných elektród od polohy srdcového zdroja, napríklad ak je srdcový zdroj umiestnený v apexe pravej komory, nájdeme najvýznamnejšie elektródy na prednej stene hrudníka. Ak sa srdcový zdroj nachádza v blízkosti bázy ľavej komory, bližšie k chrbtu pacienta, významné elektródy sa nachádzajú na prednej aj zadnej stene hrudníka. Avšak v oboch prípadoch sú významné elektródy primárne umiestnené naľavo od hrudnej kosti, bližšie k srdcu. Štúdia navyše odhalila, že významnosť elektród na hrudníku je podobná pre blízke srdcové zdroje.

Následne sme vykonali analýzu presnosti riešenia inverzného problému iba pomocou významných a nevýznamných elektród. Implementovali sme dva scenáre. Najskôr sme riešili inverznú úlohu pri postupnom začleňovaní elektród od 4 najvýznamnejších až po celý súbor, zoradených podľa ich významnosti a potom v opačnom poradí, počnúc 4 najmenej významnými elektródami. Tento prístup nám umožnil preskúmať, či špecifické polohy elektród použité pre inverzné riešenie majú vplyv na presnosť inverzného riešenia. Zistili sme, že pri použití 32 najvýznamnejších elektród bola priemerná chyba lokalizácie (LE) 26,4 $\pm$ 9,9 mm, zatiaľ čo použitie 64 elektród viedlo k priemernej LE 27,3 $\pm$ 5,6 mm a použitie všetkých elektród viedlo k LE 24,7 $\pm$ 3,7 mm. Naopak, LE bola 76,0 $\pm$ 27,6 mm pri použití 32 nevýznamných elektród a 75,9 $\pm$ 36,2 mm pri použití 64 nevýznamných elektród. Výsledky ukazujú, že riešenie pomocou podskupiny významných elektród môže viesť k porovnateľnej presnosti ako pri použití všetkých hrudníkových elektród a v určitých prípadoch bolo dokonca pozorované zlepšenie v lokalizácii. Zároveň však bolo ukázané, že použitie len nevýznamných elektród viedlo k podstatnému zhoršeniu presnosti lokalizácie.

Tieto výsledky naznačujú, že inverzný problém možno presne vyriešiť použitím menšieho počtu významných elektród. Významnosť elektród bola však stanovená na základe konkrétnej polohy zdroja srdca, ktorá nie je v klinickej praxi známa. Na vyriešenie tohto problému sme navrhli a numerickými metódami overili nový dvojkrokový algoritmus riešenia inverznej úlohy. V prvom kroku riešime inverznú úlohu za použitia všetkých elektród. V druhom kroku riešime inverznú úlohu použitím významných elektród vypočítaných pre približnú polohu srdcového zdroja získanú z prvého kroku. Pri použití 32 významných elektród zistených pre približnú polohu srdcového zdroja bola priemerná LE vypočítaná pre oba modely hrudníka 28,8 ± 14,5 mm, zatiaľ čo pri použití 64 významných elektród bola 29,5 ± 12,6 mm a 28,8 ± 11,9 mm pri použití všetkých elektród. Výsledky dvojkrokového riešenia naznačujú, že je môžné dosiahnuť podobné výsledky inverzného riešenia so zníženým počtom elektród. Tento prístup si však vyžaduje použitie všetkých elektród v prvom kroku. Okrem toho sme u niektorých pacientov pozorovali pokles presnosti lokalizácie. Napriek tomu by tento prístup mohol slúžiť ako nástroj na posúdenie spoľahlivosti inverzného riešenia. Ak sa výsledok získaný z inverzného riešenia s použitím menšej sady významných elektród výrazne líši od výsledku pri použití všetkých elektród, môže to indikovať nedôveryhodnosť prvého riešenia.

Použitím dvojkrokového riešenia inverznej úlohy je možné odhadnúť význam elektród individuálne pre každého pacienta. Zaujímavejšie je však určiť „optimálne umiestnenie elektród“ bez toho, aby bolo nutné zaznamenať povrchové potenciály všetkými elektródami ako u dvojkrokového riešenia. Aby sme identifikovali toto optimálne umiestnenie elektród, vypočítali sme významnosť elektród pre všetky možné polohy srdcového zdroja, ktoré boli rovnomerne rozložené na endoepikardiálnom modeli komôr, pre dvoch pacientov s rozdielnou geometriou hrudníka a polohou srdca. Pri 32-elektródovej konfigurácii sa významné elektródy nachádzali na prednej stene hrudníka vľavo od hrudnej kosti. Pri 64-elektródovej konfigurácii sa významné elektródy nachádzali aj na chrbte pacienta vľavo od chrbtice. Dvojkrokové inverzné riešenie prekonalo v presnosti riešenie inverznej úlohy pomocou optimálneho umiestnenia 32 eletród. Tento výsledok možno pripísať absencii elektród na chrbte pacienta v 32-elektródovej optimálnej konfigurácii. Avšak 64-elektródová optimálna konfigurácia vykazovala lepšiu presnosť v lokalizácii v porovnaní s dvojkrokovým inverzným riešením. Tieto výsledky sú obzvlášť dôležité v klinickej praxi, pretože naznačujú, že by sa mohlo používať iba 64 optimálne umiestnených elektród a nie 128 ako v súčasnosti používame na našom pracovisku. Použitie zníženého počtu elektród by účinne minimalizovalo čas potrebný na prípravu pacienta, ale tiež by mohlo zvýšiť pohodlie pacienta počas merania.

Metóda navrhnutá v tejto práci na odhad významu elektród a analýzy na dátach 8 pacientov s pacemakerom a 13 pacientov s PVC pre homogénne a nehomogénne modely hrudníka, odlišujú túto prácu od predchádzajúceho výskumu. V tejto práci sme navrhli postup nového dvojkrokového riešenia inverznej úlohy, ktorý môže byť použitý pre klinické dáta a prináša možnosť ako posúdiť spoľahlivosť nášho riešenia. V práci bol navrhnutý koncept postupu ako zistiť optimálne umiestnenie menšieho počtu elektród, ktoré by nevyžadovalo riešenie prvého kroku za použitia všetkých elektród a bolo by univerzálne použiteľné u všetkých pacientov. Výsledky tejto dizertačnej práce zdôrazňujú potenciál použitia zmenšeného počtu významných elektród na riešenie inverzného problému elektrokardiografie, čím by bolo možné implementovať pohodlnejšiu konfiguráciu elektród s menším počtom elektród. Vďaka tomu by sa mohol znížiť čas potrebný na prípravu pacienta pred povrchovým mapovaním ako aj po mapovaní. Zníženie času a zvýšenie pohodlnosti merania sú kľúčovými faktormi pre implementáciu riešenia inverznej úlohy do klinickej praxe.

Publications Related to Dissertation Thesis

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B. Ondrusova [80%], P. Tino [10%], and J. Svehlikova [10%], “A two-step inverse solution for a single dipole cardiac source,” *Frontiers in Physiology*, vol. 14, 2023. DOI: https://doi.org/10.3389/fphys.2023.1264690. Type: ADC.

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